HELSINKI SCHOOL OF ECONOMICS (HSE)
Department of Accounting and Finance

Modeling the Asset Price Dynamics

Comparison of Option-Pricing Models Using Eurex Data

Finance
Master's thesis
Jussi Paronen
Spring 2003

Approved by the Council of the Department 20 / 5 2003 and awarded the grade excellent 80 points

KTT Esa Jokivuolle KTT Jari Käppi

HELSINKI SCHOOL OF ECONOMICS
Master's Thesis in Finance
Jussi Paronen

ABSTRACT
May 9, 2003

MODELING THE ASSET PRICE DYNAMICS – COMPARISON OF OPTION-PRICING MODELS USING EUREX DATA

PURPOSE OF THE STUDY

This thesis examines different models for the asset price dynamics by comparing their ability to price and hedge options from the European derivatives market Eurex. The comparison is conducted on both index and stock options. The models included in the study are the Black-Scholes, stochastic volatility, stochastic volatility with t-distributed innovations, stochastic volatility with jumps in the price process and stochastic volatility with independent or correlated jumps in both the price and the volatility process.

The thesis examines both index and stock options. The models studied are based on Monte Carlo simulation, and therefore the applicability of simulation in option-pricing is also under scrutiny.

DATA

The data used in this study includes daily option settlement price quotes from Eurex, and daily quotes of Nokia share price and the level of DJ Euro Stoxx 50 index. Furthermore, daily observations of Euribor interest rates are used. The data period for the option and interest rate data is July 3, 2000 to June 28, 2002. For the index and stock price data the data period is March 29, 1993 to March 28, 2003.

RESULTS

The main finding of the study is that including jumps in at least the asset price process seems to be of value also in the European markets and that replacing the normal distribution of the price process innovations of the stochastic volatility model with the more fat-tailed t-distribution does not seem to generate the same effect as jumps.

The study also suggests that stock options are more sensitive to estimation and simulation error than are index options. Furthermore, when hedging options the possible errors seem to play a bigger part than in pricing options.

KEYWORDS

Derivatives, price dynamics, option pricing, hedging, stochastic process, volatility, jump-diffusion, Monte Carlo simulation

HELSINGIN KAUPPAKORKEAKOULU
Rahoituksen pro gradu - tutkielma
Jussi Paronen

TIIVISTELMÄ
9.5.2003

HINTADYNAMIIKAN MALLINTAMINEN – OPTIOHINNOITTELUMALLIEN VERTAILU EUREX-AINEISTOA KÄYTTÄEN

TAVOITTEET

Tutkielma tarkastelee eri malleja osakkeen tai indeksin hinnan kehitykselle vertaamalla mallien kykyä Eurooppalaisessa johdannaispörssissä Eurexissa listattujen optioiden hinnoitteluun ja suojaamiseen. Tutkielma tarkastelee stokastisen volatiliteetin malleja normaali- ja t-jakautunein innovaatiotermein, stokastisen volatiliteetin ja hyppydiffuusiomallin yhdistelmää sekä malleja, joissa sekä hinta- että volatiliteettiprosessiin sisältyy riippumattomia tai korreloituneita hyppyjä.

Tutkielma tarkastelee sekä indeksi- että osakeoptioita. Vertailtavat mallit ovat Monte Carlo simulointiin perustuvia, ja simuloinnin soveltuvuus optiohinnoitteluun on samalla tarkasteltavana.

AINEISTO

Tutkimusaineisto koostuu Eurexin toimittamista optioiden hinnoista, Nokian osakkeen sekä DJ Euro Stoxx 50-indeksin kursseista ja Euribor-koroista. Kaikki havainnot ovat päivittäisiä. Optio- ja korkoaineisto on peräisin ajalta 3.7.2000 – 28.6.2002, sekä osake- ja indeksiaineisto ajalta 29.3.1993 – 28.3.2003.

TULOKSET

Tutkielman tärkein tulos on, että hyppyjen salliminen ainakin hintaprosessissa parantaa mallia myös eurooppalaisen aineiston valossa. Hyppyjen korvaaminen normaalijakaumaa paksuhäntäisemmän t-jakauman käytöllä muutosten jakaumana ei näytä johtavan yhtä hyviin tuloksiin.

Tutkimusaineiston perusteella osakeoptiot ovat herkempiä estimointi- ja simulointivirheille kuin indeksioptiot. Samoin optioiden suojaamisessa mahdollisilla epätarkkuuksilla on suurempi vaikutus kuin optioiden hinnoittelussa.

AVAINSANAT

Johdannaiset, hintadynamiikka, optioiden hinnoittelu, suojaaminen, stokastinen prosessi, volatiliteetti, hyppydiffuusio, Monte Carlo - simulointi

TABLE OF CONTENTS

1 **INTRODUCTION** **7**

2 **OPTION-PRICING ISSUES** **9**

2.1 BLACK-SCHOLES MODEL 9

2.2 RISK-NEUTRAL PRICING 11

2.3 GEOMETRIC BROWNIAN MOTION 13

2.4 GENERALIZATIONS TO THE BLACK-SCHOLES MODEL 14

2.4.1 Accounting for Dividends *15*

2.4.2 American-Style and Exotic Options *16*

2.4.3 Jump-Diffusion Models with Constant Volatility *17*

2.4.4 Stochastic Interest Rates *19*

2.5 VOLATILITY SMILE 20

2.6 HEDGING 21

2.7 PRICING BY MONTE CARLO SIMULATION 24

3 **ALTERNATIVE MODELS FOR THE ASSET PRICE PROCESS** **27**

3.1 STOCHASTIC VOLATILITY MODELS 27

3.1.1 Volatility as a Diffusion Process *28*

3.1.2 Adding the Jump Component *31*

3.2 AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY 33

3.2.1 Some ARCH Models *34*

3.2.2 ARCH Models in Option Pricing *37*

3.3 CONSTANT ELASTICITY OF VARIANCE AND THE IMPLIED TREE APPROACH 40

3.4 SUMMARY OF THE MODELS 42

4 **DATA AND METHODOLOGY** **44**

4.1 DATA 44

4.2 ESTIMATION OF MODEL PARAMETERS 54

4.3 PRICING PERFORMANCE 58

4.4 DYNAMIC HEDGING PERFORMANCE 58

5 **RESULTS** **60**

5.1 ESTIMATES OF THE MODEL PARAMETERS 60

5.2 PRICING PERFORMANCE 64

5.3 DYNAMIC HEDGING PERFORMANCE 69

6 **CONCLUSIONS** **75**

7 **REFERENCES** **78**

8 **APPENDICES** **86**

LIST OF FIGURES

FIGURE 1, EXAMPLE OF A COX-ROSS-RUBINSTEIN TREE 13
FIGURE 2, EVOLUTION OF OPTION-PRICING MODELS 42
FIGURE 3, PERFORMANCE OF THE DJ EURO STOXX 50 INDEX 46
FIGURE 4, PERFORMANCE OF THE NOKIA STOCK PRICE 46
FIGURE 5, HISTORICAL DAILY NET RETURNS OF THE DJ EURO STOXX 50 INDEX AND NOKIA STOCK. 47
FIGURE 6, EURIBOR CURVES AT DIFFERENT POINTS IN TIME 48
FIGURE 7, MONEYNESS DISTRIBUTIONS OF OPTIONS ON THE DJ EURO STOXX 50 INDEX, JULY 2000 TO JUNE 2002 49
FIGURE 8, MONEYNESS DISTRIBUTIONS OF OPTIONS ON THE NOKIA STOCK, JULY 2000 TO JUNE 2002.... 49
FIGURE 9, IMPLIED BLACK-SCHOLES VOLATILITIES OF AT-THE-MONEY OPTIONS ON DJ EURO STOXX 50 INDEX 51
FIGURE 10, IMPLIED BLACK-SCHOLES VOLATILITIES OF AT-THE-MONEY OPTIONS ON NOKIA STOCK 52
FIGURE 11, THE AUTOCORRELATION FUNCTIONS OF THE BS IMPLIED VOLATILITIES 53

LIST OF TABLES

TABLE 1, PROPERTIES OF THE HISTORICAL DAILY NET RETURN DISTRIBUTIONS 48
TABLE 2, PARAMETER ESTIMATES FOR OPTIONS ON DJ EURO STOXX 50 62
TABLE 3, PARAMETER ESTIMATES FOR OPTIONS ON NOKIA STOCK 63
TABLE 4, OUT-OF-SAMPLE PRICING ERRORS, DJ EURO STOXX 50 INDEX 65
TABLE 5, OUT-OF-SAMPLE PRICING ERRORS, NOKIA STOCK 66
TABLE 6, OUT-OF-SAMPLE HEDGING ERRORS, DJ EURO STOXX 50 INDEX 70
TABLE 7, OUT-OF-SAMPLE HEDGING ERRORS, NOKIA STOCK 71

ABBREVIATIONS

ACF	Autocorrelation Function
AR	Autoregressive
ARMA	Autoregressive Moving Average
ARCH	Autoregressive Conditional Heteroskedasticity
ATM	At-The-Money; asset price/strike price ≈ 1
BS	Black-Scholes model for option price
CEV	Constant Elasticity of Variance
CIR	Cox-Ingersoll-Ross model for interest rates or volatility
GARCH	Generalized ARCH
GBM	Geometric Brownian Motion
GMM	Generalized Method of Moments
EGARCH	Exponential GARCH
EMM	Efficient Method of Moments
EMS	Empirical Martingale Simulation
LRNVR	Locally Risk-Neutral Valuation Relationship
ITM	In-The-Money; for call options asset price/strike price > 1
MCMC	Monte Carlo Markov Chain
ML	Maximum Likelihood estimation method
NGARCH	Non-linear Asymmetric GARCH
OTM	Out-of-The-Money; for call options asset price/strike price < 1
OU	Ornstein-Uhlenbeck process
SDE	Stochastic Differential Equation
SSE	Sum of Squared Errors
SV	Stochastic Volatility
SVCJ	Stochastic Volatility with Correlated Jumps in price and volatility processes
SVIJ	Stochastic Volatility with Independent Jumps in price and volatility processes
SVJ	Stochastic Volatility with Jumps in the price process
SVT	Stochastic Volatility with T-distributed innovations in the price process

1 Introduction

In this thesis different ways to value options are studied and compared. In order to be able to price an option, we must know or assume what the distribution of the payoffs of the option is. Like Cox and Ross (1976, p. 154) write, "if we know the cumulative probability distribution of the stock process we can value the option". There are of course many ways to model this distribution, the most famous of which is the option-pricing model of Black and Scholes (1973). However, academics have known already since the 1970s (e.g. Merton, 1973 and Black, 1976) that the assumptions made by Black and Scholes simplify the real world too much and therefore the resulting option prices are not exactly right.

Naturally, as large sums of money are at stake in the derivatives market, several improvements to the Black-Scholes (from now on denoted as BS) model have been tried over the last 30 years. Still, in spite of all the models introduced, the BS-model is even today used broadly. Maybe it is because ultimately the choice is between misspecified models, as Bakshi et al. (1997, p. 2004) discuss. It can be said with certainty that the perfectly specified option-pricing model is bound to be too complex for practical use. Still one would like to believe that at least some improvement to the documented biases of the BS-model can be reasonably implemented, and this has indeed been the subject of quite a few empirical papers in the past. For example Bates (1996, 2000), Bakshi et al. (1997, 2000) and Andersen et al. (2002) find evidence that additional features and more general assumptions can significantly improve the option pricing performance of the models, but yet manage to keep their models sufficiently simple for practical use.

Despite the large number of papers published in this field, they all tend to be conducted on data from the American market. Even though it is natural to assume that the same rules apply in the European market as well, there is not much empirical evidence of this, which is why our focus is on the European markets. Additionally, this thesis covers some more recent models presented by Duffie et al. (2000) that, except for the paper by Eraker et al. (2002), have received little attention in empirical studies.

Therefore it is the goal of this study to compare some of the most important models for the asset price dynamics in the context of option pricing, and to do this on data obtained from the European derivatives market Eurex. The different models are compared in terms of their in-sample and out-of-sample pricing performance as well as out-of-sample hedging performance.

The remainder of this thesis is organized as follows: in section 2 the basic models for option pricing and modeling asset price dynamics as well as some generalizations of the basic model are discussed. The related issues of the volatility smile, hedging the options and pricing them through simulation will also be covered. In section 3 the focus is on different ways to model the asset price dynamics. This section also defines the models studied in the empirical part of the thesis, formed by sections 4 and 5. Finally, section 6 concludes.

2 Option-Pricing Issues

In this section we will start with the basic option pricing theories, namely those of Black and Scholes (1973) and a discrete-time pricing method, known as the risk-neutral pricing formula. We will also discuss the assumptions of the Black-Scholes model and discuss how these assumptions might be relaxed to further generalize the model and to improve pricing efficiency.

2.1 Black-Scholes Model

The biggest step in the history of option pricing was taken in 1973, when Black and Scholes published their path-breaking paper in which they derived an analytic pricing formula for the price of an European-style option, i.e. an option that can only be exercised at maturity. The theory is based on no-arbitrage principle and, unlike previous attempts to price options, the resulting formula does not depend on investor preferences or other subjective variables.

The "ideal conditions" assumed by Black and Scholes (1973, p. 640) are

i) The short-term interest rate is known and constant through time.

ii) The asset price follows a random walk in continuous time with a variance rate proportional to the square of the asset price. Thus the distribution of possible asset prices at the end of any finite interval is lognormal. The variance rate of the return of the stock is constant.

iii) The asset pays no dividends or other distributions.

iv) The option is European-style.

v) There are no transaction costs in buying or selling the asset or the option.

vi) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.

vii) There are no penalties for short selling.

The Black-Scholes (BS) formula has later been studied extensively by various authors, and several of these assumptions have been relaxed, as will be discussed later in this thesis. The original BS formula, however, took the form

$$C(S,t,T,r,K,\sigma) = SN(d_1) - Ke^{-r(T-t)}N(d_2) \tag{2.1}$$

where

$$d_1 = \frac{\log\left(S/K\right) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = d_1 - \sigma\sqrt{T-t},$$

N(•) is the standard cumulative normal distribution,
K is the exercise price,
S is the price of the underlying asset,
r is the risk-free short rate,
T is the expiration time,
t is the time, such that T-t is the time to maturity, and
σ is the instantaneous volatility of the asset price.

To value a put option, the formula has to be modified a little, but the principle remains the same. Also the so-called put-call parity, which is a simple arbitrage relationship between European put and call options, can be used for obtaining put prices from call prices. The put-call parity can be written as

$$C(0) - P(0) = S(0) - Ke^{-rT} \tag{2.2}$$

where C is the price of an European call, P the price of an European put, S the price of the underlying asset, K the exercise price, r the risk-free rate and T the time to maturity. For proof, see Jarrow and Turnbull (2000, pp. 79-81).

2.2 Risk-Neutral Pricing

The idea of risk-neutral pricing emerged in the work of Black and Scholes (1973), but the formal concept can be traced back to Rubinstein (1976), Cox and Ross (1976), Ross (1978) and Brennan (1979), after which it was formalized by Harrison and Kreps (1979). Today, risk-neutral pricing is an integral part of the modern finance theory and forms the basis e.g. for the simulation method for pricing options, which is used also in this thesis. The concept of risk-neutral pricing is based on no arbitrage and complete markets, so that it can be shown that unique, positive state prices ψ_s[1] exist. An alternative way to represent state prices is to use the so-called risk-neutral probabilities. To get there we start by writing the (single period) pricing formula as

$$P = \sum_{s=1}^{S} d_s \psi_s , \tag{2.3}$$

where d_s is the payoff in state s and ψ_s the corresponding state price. We define

$$\psi_0 \equiv \sum_{s=1}^{S} \psi_s \tag{2.4}$$

and

$$q_s \equiv \psi_s / \psi_0 . \tag{2.5}$$

where q_s is called the risk-neutral probability. It is an artificial measure, which acts like a probability (all q:s are positive and they sum to one). Using the q:s as probabilities we can now write (2.3) as

$$P = \psi_0 E^Q(d) \tag{2.6}$$

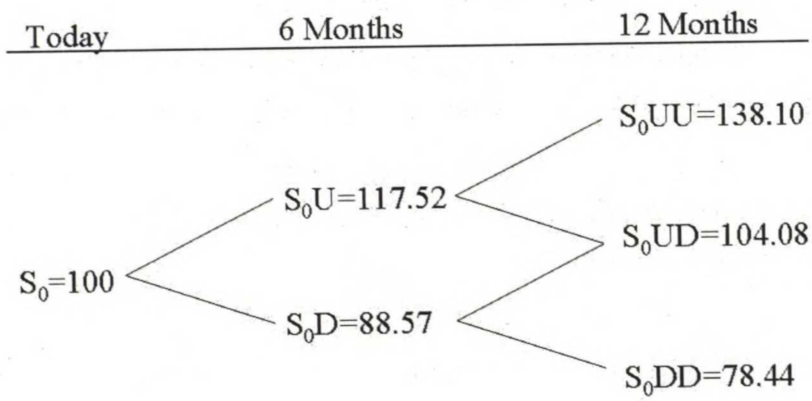
[1] State price is the price of a contingent security (or 'an Arrow-Debreu security' or 'an elementary state security'), i.e. a security that pays one in a given state of the world and zero otherwise. The state price can thus be interpreted as the discount factor from this specific state to time zero.

Where d is the vector of d_s:s and E^Q is the expected value using the q:s as probabilities. As opposed to the true probability measure P, Q is therefore called the risk-neutral probability measure, or often also the equivalent martingale measure. Since ψ_0 is the sum of all state prices ψ_s, $s \in [1,S]$, it is actually a price of a security that pays 1 in every state, or in other words, the price of a risk-free bond. So, by definition, $\psi_0 = 1/(1+r)$, where r is the risk-free rate. Hence, the pricing formula can be written as

$$P = \frac{E^Q(d)}{1+r} \tag{2.7}$$

More details and proofs about linear pricing, state prices and risk-neutral prices can be found in Luenberger (1998, Chapter 9, pp. 228-259) and of course in the original references, especially Harrison and Kreps (1979). The concept can also easily be extended into a recursive model of several periods.

The risk-neutral is at its simplest when applied in the binomial lattice context, first presented by Cox, Ross and Rubinstein (1979). They modeled the price process of an asset such that in any period the price can move only up (u) or down (d), the (relative) movements being equal in all periods. This way a recombining tree is formed (ud = du). A numerical example of a Cox-Ross-Rubinstein tree can be seen in Figure 1 below. The property that the tree is recombining is valuable in terms of making the calculations less burdensome. For an n-step recombining binomial lattice there will only be n+1 nodes, whereas for a tree that is not recombining, the number of nodes would be 2^n. That is why recombining lattices are usually preferred in modeling, if only they can be applied to the problem at hand.



The up factor $U = 1.1752$
The down factor $D = 0.8857$
$R = e^{r\Delta t} = 1.0304$ for Δt = six months

Figure 1, Example of a Cox-Ross-Rubinstein Tree

(Jarrow & Turnbull, 2000, p. 119)

From this thesis' perspective the most important consequence of risk-neutral pricing is that the price process, whether including stochastic volatility, GARCH property or jumps, has to be modified accordingly. The differences in true and risk-neutral distributions have to be taken in account, so the parameters directly observable under the true probability methods are not directly applicable as such. In changing the probability measure a powerful tool is Girsanov's theorem (see e.g. Duffie, 2001, p. 111 and Jarrow & Turnbull, 2000, pp. 245-246), using which the risk-neutral process can be derived.

2.3 Geometric Brownian Motion

By far the most common basic model for asset price dynamics, i.e. the price evolution over time, is the geometric Brownian motion (GBM), where the asset price is lognormally distributed and the volatility remains constant over time. The underlying asset of an option is assumed to follow this process for example under the Black and

Scholes (1973) assumptions. GBM is an Ito process (see Duffie, 2001, p. 86) of the form

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{2.8}$$

where S_t is the asset price at time t, μ is the drift, σ is the instantaneous volatility of S (often also referred to as the diffusion of S) and W is the standard Brownian motion.[2]

Then, based on Ito's lemma (see e.g. Luenberger, 1998, p. 312, Duffie, 2001, p. 87 or Jarrow & Turnbull , 2000, pp. 213-215) and Girsanov's theorem, in a risk-neutral world the process can also be written in the following form:

$$d\log S_t = \left(r - \frac{\sigma^2}{2}\right)dt + \sigma dW_t \tag{2.9}$$

2.4 Generalizations to the Black-Scholes Model

In the literature, many of the BS assumptions have been considered as oversimplifying reality, but the assumption of constant volatility has been seen as especially restricting. It has been commonly accepted since the work by Mandelbrot (1963) and Fama (1965) that uncertainty, in terms of variances and covariances, does not remain constant but varies through time. For example, Mandelbrot states that "…large changes tend to be followed by large changes – of either sign" (p. 418) so, in other words, volatility tends to cluster. There is no consensus about the fundamental reasons behind this phenomenon, but some discussion can be found in Schwert (1989) and Campbell and Hentschel (1992), among others. As the volatility-assumption can be seen as the most crucial assumption affecting the tails of the return distributions and also otherwise as the most central assumption, it will be discussed in more detail in section 3. In that

---

[2] Duffie (2001, p. 83) defines the standard Brownian motion W (also known as Wiener process) as follows: (a) W_0=0 almost surely; (b) for any times t and s, s > t, W_s-W_t is normally distributed with mean zero and variance s-t; (c) for any times $t_0,\ldots,t_n$ such that $0 \leq t_0 < t_1 < \ldots < t_n < \infty$, the random variables W_{t0}, $W_{t1}-W_{t0},\ldots,W_{tn}-W_{tn-1}$ are independently distributed; and (d) the sample path W is continuous.

section of the paper the models studied in the empirical part of this thesis will be presented and the assumptions made for those models are also discussed further. In this section we limit ourselves to discussion on other kinds of typical extensions to the BS model. These include the jump-diffusion models with constant volatility and models with stochastic interest rates, as well as the more traditional problems with the BS model, namely relaxing the assumption of zero dividends and how to price other than European-style options. In addition to those generalizations, BS can also be generalized to price options on futures or forwards, but this generalization will not be discussed here any further, as it is of little relevance to us[3].

2.4.1 Accounting for Dividends

The no-dividend assumption was one of the first ones to be addressed in the literature, already by Merton (1973). If a known (discreet) dividend d is paid at time t_1, $t < t_1 < T$, the distribution of the asset price is no longer lognormal, causing another assumption to be modified. After this modification the distribution is assumed a "displaced lognormal distribution" prior to date t_1. This implies that the smallest possible value for the asset price at time t_1 is d (whereas in a lognormal distribution it would be zero). Using the risk-neutral valuation procedure, the price of an European call option at time $t = 0$ can then be calculated by the modified BS formula

$$C(0) = H(0)N(d_1) - Ke^{-rT}N(d_2) \tag{2.10}$$

where the only difference to the standard BS equation is that asset price S(0) is replaced by $H(0) \equiv S(0) - de^{-rt_1}$, so the present value of the dividend is subtracted from the asset price. H(0) is also used instead of S(0) in calculating d_1 and d_2. For more details, see Jarrow & Turnbull (2000, pp. 251-253).

It is also possible to value European options assuming that the asset pays a continuous dividend yield d_y. This yield is modeled constant, which implies that the monetary

---

[3] An interested reader should see e.g. Jarrow & Turnbull (2000, pp. 261-265)

dividend is random (as the dividend is a constant proportion of a random variable, the asset price). This assumption often proves useful when pricing currency options, index options, commodity options or options on futures. This assumption will lead to modified risk-neutral drift of $r-d_y-\sigma^2/2$, which in turn leads to pricing formula

$$C(0) = e^{-d_y T} S(0) N(d_1) - K e^{-rT} N(d_2) \tag{2.11}$$

where d_1 and d_2 are as in the original BS formula. To put it another way, only the asset price is 'discounted' with the constant dividend yield d_y. Again, more details can be found e.g. in Jarrow and Turnbull (2000, pp. 258-261, 271).

There are of course also other ways to model the dividends, and the problem will only escalate when pricing American options. Some ways to price American options with dividends are presented in Jarrow & Turnbull, (2000, pp. 255-258). If these simplifying assumptions about dividends can not be made, then the BS formula can not be used in pricing these options, but other methods have to be applied.

2.4.2 American-Style and Exotic Options

The assumption iv), that the option is European-style, is another one addressed by Merton (1973). He reasoned that an American option must always be at least as valuable as an European one, because it can always be exercised only at maturity. Hence, an American option, denoted C_{AM}, is always a European option C plus an additional feature, an option to exercise earlier than at maturity. The European call in turn is at least as valuable as a forward contract on the underlying with the delivery price equal to the exercise price of the option, because the option is actually the forward contract plus an additional feature of not having to exercise if the payoff is negative. These relations combined, we can write the inequality $C_{AM}(0) \geq C(0) \geq \mathrm{Max}[0, S(0)-Ke^{-rT}]$. The difference between the prices C_{AM} and C is therefore never bigger than the time value of the American option. Time value is the value of not exercising the option but waiting, and is defined as $C_{AM}(0)-[S(0)-Ke^{-rT}]$. Because of that relation, when there are no dividends, it is never optimal to exercise an American call option before maturity. Therefore, as it is always the choice of a rational investor to exercise the

American option exactly as a European option would be exercised, the values of these options must also be equal. Therefore, the original BS formula can be used to price American call options if there are no dividends. Having said that, with put options the values of American- and European-style options are not always equal. The values can differ, because if the price of the underlying asset falls low enough, the maximum payoff, $K-S_t$, can be reached already before maturity. The simplest example would be S falling to zero, so it is not possible for K-S to get any bigger. Hence the maximum, K, is already reached. Then it would be optimal to exercise now rather than in the future, to capture the time value of the profit as well. In the case of dividends it can be optimal to exercise also a call option just prior to the ex-dividend date, if the dividend is large enough. In these cases the option price can not be calculated by the BS formula, but alternative methods, such as the binomial lattice, must be used.

There are also other types of options than European or American. These are often called exotic options. Some examples of these are Asian options, in which the payoff depends on the average price of the underlying during the period of the option, compound options, which are options on options and knockout options, that terminate with zero value once the price of the underlying reaches a specified point. There are also several other kinds of exotic options, and only imagination can limit the development of new ones. Some of the most common exotic options are briefly described by Luenberger (1998, pp. 368-371). In general exotic options can not be priced by the BS formula or modified versions of it, but other kinds of pricing methods have to be applied.

2.4.3 Jump-Diffusion Models with Constant Volatility

Yet another way to generalize the BS-assumptions of the price process of the underlying asset, in addition to non-constant volatility, is allowing the asset price process to be discontinuous, i.e. allowing it to jump at times. An extreme version of this would be that the movement of the asset price is caused purely by jumps, not at all through a diffusion process such as GBM. An example of models of this kind can be found for instance in Cox and Ross (1976). A more common version of jump models is, however, the so-called jump-diffusion model, where the model also has a jump component in it but that follows a diffusion process the rest of the time. Merton (1976) was the first author to present a model of this type. In his model the volatility of the diffusion process –i.e. the continuous part of the price process – remains constant.

The aim in jump-diffusion models is fundamentally the same as in models with non-constant volatilities, to have a more fat-tailed distribution for stock returns - observed empirically by for example Fama (1965) – and in this way to obtain more accurate and realistic option prices. Merton (1976) suggested that this way the tendency for deep-in-the-money, deep-out-of-the-money and short-maturity options to sell for more than their BS value could be explained. As Merton (1976, p. 128) discusses, just as the standard Brownian motion is the natural prototype for the continuous component of the asset price process, the natural prototype for the discontinuous component, i.e. the jump component, is a "Poisson-driven" process. He describes this component as follows: it is assumed that the events (jumps) are i.i.d. and the probabilities of an event taking place during a time interval h (where h can be arbitrarily small) are drawn from a Poisson distribution. With these assumptions the price process can be written as

$$\frac{dS}{S} = (\mu - \lambda k)dt + \sigma dW + dq \tag{2.12}$$

where μ is the instantaneous expected return of the stock, σ is the instantaneous volatility conditional on the Poisson event not occurring, W is the standard Brownian motion, q(t) is the independent Poisson process, λ is the mean number of events taking place per unit time and $k \equiv E(Y-1)$ where (Y-1) is the random variable percentage change in the stock price if the jump takes place. Note that if $\lambda = 0$, then $dq \equiv 0$ and the process follows GBM and is equivalent to the BS model. Now the process can be rewritten as

$$\begin{aligned} \frac{dS}{S} &= (\mu - \lambda k)dt + \sigma dW && \text{if the jump does not occur} \\ &= (\mu - \lambda k)dt + \sigma dW + (Y-1) && \text{if the jump occurs,} \end{aligned} \tag{2.13}$$

where, with probability one, no more than one jump can take place in an instant and, should the jump occur, (Y-1) is the impulse function producing a finite jump in S to SY.

Merton (1976) assumed, in addition to the independent lognormally distributed jumps, that jump risk is diversifiable. This assumption has later been relaxed and results by e.g. Bates (1991) indicate that Merton's model with modified parameters is still relevant even under nondiversifiable jump risk.

Das and Sundaram (1999) compared in their paper the model with jumps but constant volatility against the model with stochastic volatility but no jumps, and found that in both models some patterns fundamentally inconsistent with the observed data can be found. Of these two models the one with stochastic volatility performed better, especially when time to maturity gets longer. In the short term the effect of jumps is more significant. Based on their results we will not be testing the jump-diffusion model of Merton (1976) as such in this study. However, we will be adding a jump component to the price and even the volatility process much in a similar way as Merton added it to GBM. These combinations of stochastic volatility and jump-diffusion models will be discussed in more detail in section 3.1.2. The findings of Bakshi et al. (1997, 2000) and Bates (2000) show that even though the stochastic volatility is found to be the most significant factor in explaining the pricing biases of the BS model, allowing the asset price to jump does also have a significant effect on the pricing performance of the model. Recently this result has been further emphasized by Andersen et al. (2002), Pan (2002) and Eraker et al. (2002).

2.4.4 Stochastic Interest Rates

An exhaustive number of papers concerning the term structure of interest rates has been written over the years and this is indeed a very important research area in financial economics. As interest rates are also a factor affecting option prices, relaxing the BS assumption of constant interest rates has been studied in several papers. In option pricing literature, the most commonly used model - though by no means the only one - for the interest rate dynamics is the Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985), which is based on an intertemporal general equilibrium asset pricing model. The (single factor) CIR model for the dynamics of the interest rate r is

$$dr = \kappa(\theta - r)dt + \sigma_r \sqrt{r} dW_1 \tag{2.14}$$

where κ is the mean-reversion parameter, θ is the drift, σ_r is the volatility of r and W_1 is a one-dimensional Wiener process.

In option pricing, allowing the interest rates to be stochastic would mean a third stochastic differential equation (SDE).[4] Usually the assumption is that the interest rate process is uncorrelated with the rest of the economy, i.e. the processes for the asset price and for the volatility.

Despite the great importance of term structure models in finance and economics, it has been found by e.g. Bakshi et al. (1997, 2000) that in option pricing adding the third differential equation for interest rates does not add much to option pricing models with stochastic volatility. They found that even for long-term options, which one would assume to be more sensitive to interest rates, adding stochastic interest rates does not improve the performance of the model. Therefore, even in spite of Bailey and Stulz (1989) and Amin and Ng (1993), who argue that in an equilibrium stochastic volatilities must imply also stochastic interest rates, models with stochastic interest rates will not be tested empirically in this thesis.

2.5 Volatility Smile

This nonparametric approach of testing an option-pricing model was presented by Rubinstein (1985). The idea is to calculate the implied volatilities (the volatility with which the model price is equal to the market price) out of observed option prices by the model to be tested, e.g. the BS. Then these implied volatilities are compared against each other across times to maturity or across moneyness, i.e. the ratio of asset price and the exercise price. Based on this ratio, the option is said to be in-the-money (ITM) if the asset price is greater than the strike price, so the ratio S/K is bigger than one. Option is

---

[4] If stochastic volatility is assumed, which is usually the case when stochastic interest rates are being considered. The author has no knowledge of option pricing models, in which the volatility is assumed constant but interest rates are stochastic.

said to be at-the-money (ATM) if the ratio is approximately one and out-of-the-money (OTM) if the ratio is smaller than one. Often these patterns are presented graphically. The word 'smile' comes from the typical shape of the graphical presentation, namely that the volatility is lower when the moneyness is close to one, i.e. the option is approximately at the money. Another typical shape is a 'smirk', in which the volatility decreases as the option goes deeper in the money[5]. In this thesis the word 'volatility smile' will be used for all patterns like this, both for smiles and smirks.

This phenomenon is something we would not like to see, as there is of course supposed to be just one volatility for one asset at any given time point, just as there is just one asset price. That is why the volatility smile is a phenomenon often regarded as evidence of a misspecified model.

2.6 Hedging

The idea of hedging an option by replication is to construct a synthetic option, i.e. a portfolio such that it behaves similarly to the option to be hedged. Then, if you buy the option and sell the portfolio - or vice versa - all changes in their values offset each other and the end value is deterministic and therefore, in this sense, riskless. In order to achieve this the portfolio must take into account all the underlying risk factors. For example, to hedge the risk related to the price movements of the underlying asset one must, for each hedged option, have X_S (the calculation of which will be discussed later, in section 4.4) units of the underlying asset. As a result, when S changes one unit, both the value of the option and of the hedging portfolio change by the same amount, though in opposite directions. However, usually constructing a perfect hedge is not possible or is too expensive, so some risk remains for the investor to carry.

Hedging an option by replication is one of the most important practical issues related to the topic of this thesis. Often academics assume that the most important thing for a pricing model is to *price* an option as accurately as possible. This is of course also

---

[5] Note that we are discussing call options. For put options the figures and patterns are very similar, but as the S/K ratio increases the put goes deeper out of the money.

important, as already implied by the name "pricing model", but in practice the price quote is often available from the market. Naturally this does not always hold true, for instance with OTC-products that are tailored to the customers needs. But the hedge ratios - meaning the number of each hedging asset in the hedging portfolio and calculated using the partial derivatives of the option price with respect to the underlying market or model parameters such as the asset price or initial volatility - have to be calculated from the model every time, as they are never directly available at the market. The hedging performance is therefore an important measure when comparing alternative pricing models. (Reiss & Wystup, 2001)

Hedging can be done in many ways, depending on which risks we wish to control and how complex we allow the hedging portfolio to be. Bakshi et al. (1997) divide their discussion in two parts, namely single-instrument hedging and delta-neutral hedging. The single-instrument hedge, as the name implies, consists only of a single hedging instrument, the underlying stock. This constraint causes that even though the hedge is relatively simple and can in some cases be the most practical hedging strategy,[6] risk factors uncorrelated with the underlying asset can not be controlled for by any position in the stock. For this reason we will concentrate only in the delta-neutral hedging in this thesis.

If one can use any instruments necessary to create a perfect hedge, a delta-neutral hedge as defined by Bakshi et al. (1997, pp. 2036-2042) is constructed. This differs from the single-instrument hedge when there are more risk factors than just the underlying asset, for instance volatility, interest rate or jump risk. Note that one needs as many hedging instruments as there are sources of risk. In our case that means that for the BS model the underlying asset is enough, whereas for the models with changing volatility a second call option is needed to control for the volatility risk. With jump-diffusion models we follow Bakshi et al. (1997) and Merton (1976) and only construct a partial hedge in which only diffusion risks are hedged but jump risk is left uncontrolled for. This is because a perfect hedge, i.e. a hedge that also covers jumps with stochastic sizes, may not be feasible. This has been previously discussed by Bates (1996a) and

---

[6] Bakshi et al. (1997) argue that single-instrument hedge can be rendered practical by model misspecification and transaction costs.

Cox and Ross (1976), in addition to the already mentioned Merton (1976) and Bakshi et al. (1997) As we do not allow for stochastic interest rates in the models, we can not and do not have to hedge the interest rate risk. In other words, our assumption is that there is no interest rate risk.

In constructing the delta-neutral hedge we will need the partial derivatives of the option price with respect to the risk factors (underlying asset S and volatility V), which we denote, following Bakshi et al. (1997)

$$\Delta_S = \frac{\partial C}{\partial S_0} \tag{2.15}$$

and

$$\Delta_v = \frac{\partial C}{\partial V_0}, \tag{2.16}$$

where S_0 is the asset price at time zero and V_0 the instantaneous volatility at time zero.

The calculation of these derivatives is presented in Appendix A. Relevant references for computing the partial derivatives of the option price, especially in a simulation context, are for example Broadie and Glasserman (1996), Boyle et al. (1997), Glasserman and Zhao (1999) as well as Reiss and Wystup (2001).

These deltas are then used to construct the hedging portfolios. The deltas are naturally different if calculated with different models, because the derivatives are dependent on the underlying price dynamics, i.e. the pricing model. The details of the hedging portfolios will be discussed further in section 4.4 after the corresponding pricing models have been introduced.

2.7 Pricing by Monte Carlo Simulation

Using Monte Carlo simulation to price options is a method first used by Boyle (1977). The basis for the new tool had already been laid by Cox and Ross (1976), who first showed that pricing a European option is actually equivalent to knowing the risk-neutral distribution of payoffs at maturity. The idea of option pricing through Monte Carlo simulation is simple. Once we know – or assume – the risk-neutral price process for the asset price (at its simplest, GBM), we simulate the path n times so that we get a simulated distribution at maturity. Then we just take the expected value at maturity and discount it to the present time with the risk-free rate[7].

However, there are also some drawbacks in using simulation. Firstly, it is essentially only applicable for pricing European-style options, even though methods for approximating prices of other types of options have also been presented. For us this causes no problems, as in the empirical part we will only be studying European options. Secondly, simulation can be burdensome in terms of computer time, as typically a large amount of simulation trials is needed. With few trials the results obtained are not reliable, as the accuracy increases with the number of simulation runs. In general, the expected error decreases with the number of trials n by the factor $1/\sqrt{n}$, so one more digit of accuracy requires a hundred times as many trials. Usually at least thousands, if not tens of thousands, of trials are required in order to obtain satisfactory accuracy. Even if simulation can be costly in terms of computer time, it is still often used as it is relatively reliable and it provides flexibility and ease of programming. In some cases, depending on e.g. the underlying price process or the path dependency of the payoff, simulation can be practically the only feasible method of pricing a security.

To improve the efficiency of simulation, various variance reduction methods are often used. The two most common are the antithetic variable method and the control variate method (for descriptions of these and some other common variance reduction methods, see Boyle et al., 1997 or Clewlow & Strickland, 1998). Even though both of these variance reduction methods are widely used and have their good sides, neither of them

---

[7] For a more detailed description of the general simulation process, see Boyle et al. (1997) or Clewlow and Strickland (1998).

will be used in this thesis. Instead we will be using an Empirical Martingale Simulation (EMS) method, presented by Duan and Simonato (1998).

The Empirical Martingale Simulation method is a simple modification to the standard Monte Carlo simulation procedure. It ensures that the price estimated by simulation satisfies the rational option pricing bound, i.e. $\hat{C}(\tau, n) \geq \max(S_0 - Ke^{-r\tau}, 0)$, where $\hat{C}(\bullet)$ is the option price obtained by simulation, τ is time to maturity, n is the number of simulations and S_0-$Ke^{-r\tau}$ is the payoff of an option exercised immediately. As illustrated by Duan & Simonato (1998, p. 1220), this bound is sometimes violated when using standard simulation methods. The reason why the bound is not violated when using EMS, is that the theoretical martingale property is satisfied. In other words, for any T ≥ t ≥ 0 the discounted asset price is a Q-martingale:

$$E^Q\left[e^{-rT} S(T) \mid \phi_t\right] = e^{-rt} S(t) \tag{2.17}$$

where $E^Q(\bullet)$ denotes the expectation operator under the risk-neutral measure Q and ϕ_t denotes the information set at time t.

When using the EMS procedure, the properties discussed above are satisfied by construction. When using the EMS, the standard Monte Carlo sample $\{s_{1,t}, s_{2,t}, \ldots, s_{n,t}\}$, at time t with n simulation runs, is converted to $\{s^*_{1,t}, s^*_{2,t}, \ldots, s^*_{n,t}\}$ using the following transformation:

$$s^*_{i,t} = \frac{S_0 e^{rt}}{\overline{S}_{n,t}} s_{i,t} \tag{2.18}$$

where

$$\overline{S}_{n,t} = \frac{1}{n} \sum_{i=1}^{n} s_{i,t} \,. \tag{2.19}$$

As the EMS ensures the satisfaction of the rational bounds, it yields a substantial reduction in Monte Carlo errors. Note that the theoretical martingale property is satisfied by the adjusted sample. Let t_0 be the present time, then

$$E^Q\left[e^{-rt}S(t)\middle|\phi_0\right]=e^{-rt}\frac{1}{n}\sum_{i=1}^{n}s_{i,t}^{*}=e^{-rt}\frac{1}{n}\sum_{i=1}^{n}\frac{S_0e^{rt}}{\overline{S}_{n,t}}s_{i,t}$$

$$=\frac{S_0}{\overline{S}_{n,t}}\frac{1}{n}\sum_{i=1}^{n}s_{i,t}=S_0 \tag{2.20}$$

for any t (Duan et al., 2001).

Even though standard variance reduction techniques could easily be coupled with the EMS, Duan and Simonato (1998, pp. 1227-1229) argue based on their empirical results that it is actually the most efficient way, measured by the ratio of the standard root mean squared error and computation time, to use EMS without additional variance reduction methods. Duan and Simonato (1998, p. 1222, Proposition 1.) also show that the EMS preserves consistency, i.e. convergence to the theoretical value, under fairly general conditions and that this consistency is irrespective of the pricing framework used, i.e. whether the simulated model is BS, GARCH or something else. The EMS has, however, one drawback. Because the EMS adjustment creates dependency among the sample paths, the standard error of the price estimate is not readily available as with standard simulation methods. Duan, Gauthier and Simonato (2001) have tackled this problem in a follow-up paper, showing that the distribution of the estimate is asymptotically normal. Their results allow the calculation of confidence intervals for an individual option price estimate.

3 Alternative Models for the Asset Price Process

As mentioned in Section 2.4, volatility has been commonly considered the most restricting assumption of the BS model, and therefore it is natural that several models incorporating changing volatility to the asset dynamics have been introduced over the years. Therefore modeling the volatility process is the main theme of this chapter, although jump diffusion models are also included in the discussion. In this chapter alternative models with stochastic volatility processes and also jumps in price and even the volatility processes will be presented. These are the models to be examined empirically later in the thesis. Also other ways to model the time-variant volatilities are discussed in this chapter. These include the ARCH-type models and modeling the volatility as a deterministic function of the asset price and time. These models, however, will not be included in the empirical analysis.

What makes any model with stochastic volatility, jumps and/or stochastic interest rates different from the basic models of Black and Scholes (1973) or Merton (1973), is that one has to make additional assumptions to be able to price the option. Standard practices for pricing the volatility risk have typically involved either assuming that the risk is non-systematic and therefore has zero price, or by imposing a functional form on the risk premium with extra parameters to be estimated from observed option prices. These risk premia can potentially make the risk-neutral distribution and the true distribution of the underlying asset price differ from each other more than in basic models. (Bates, 1996b, pp. 572-573)

3.1 Stochastic Volatility Models

In this section we will be discussing stochastic volatility models and their evolution during the last fifteen years. Then we will generalize the models to combine the jump-diffusion and SV models. In doing so, we will be defining the models to be examined in the empirical part of the thesis.

3.1.1 Volatility as a Diffusion Process

The first option pricing models with stochastic volatility were presented in the late eighties, as several papers (some of the most important ones being Hull & White, 1987, Johnson & Shanno, 1987 and Wiggins, 1987) were published within a short time span. The idea was to allow volatility (or variance) to follow a diffusion process similar to the asset price process.

In different papers the model used for the volatility process has been different, most often either an Ornstein-Uhlenbeck (OU) process for the logarithm[8] of instantaneous conditional volatility or a square root process similar to the CIR interest rate model of Cox et al (1985), presented also in section 2.4.4. The OU process can be thought of as the continuous-time equivalent of an AR(1) process. According to Bates (1996b) there has been little research as to what is the correct specification of the volatility process and both of the models are consistent with the results that volatility is mean-reverting (which has been documented by e.g. Merville & Pieptea, 1989). The CIR model for volatility, as it is sometimes called, used by most empirical papers, including. Bakshi et al. (1997, 2000), implies that we are assuming the following price and volatility processes under the risk-neutral measure:

$$d\log S_t = \left(r - \frac{V(t)}{2}\right)dt + \sqrt{V(t)}dW_{t,s} \tag{3.1}$$

$$dV(t) = [\kappa(\theta - V(t))]dt + \sigma_v\sqrt{V(t)}dW_{t,v} \tag{3.2}$$

where (3.1) is identical to (2.9), only with the variance σ^2 written as V(t) and being time-dependent, and the standard Brownian motion W_t indexed $W_{t,s}$ to differentiate between the asset price process and the volatility process. In (3.2), θ and κ are constant parameters for the drift and the mean-reversion parameter, respectively, just as in the interest rate model (2.14), but now for the volatility process. σ_v is the volatility of volatility and $W_{t,v}$ is a standard Brownian motion for the volatility process. We allow

---

[8] To ensure the non-negativity of volatility

$W_{t,s}$ and $W_{t,v}$ to be correlated with the correlation efficient ρ. In practice, this is implemented by defining (see Clewlow & Strickland, 1998, p. 109)

$$\begin{aligned} W_{t,s} &= z_1, \\ W_{t,v} &= \rho z_1 + \sqrt{1-\rho^2} z_2 \end{aligned} \tag{3.3}$$

with independent random variables z_1 and z_2. Typically, but not necessarily, z_1 and z_2 would be defined as standard normal variables.

Now this raises the question whether allowing for correlation between the two processes really is necessary. If it was not, we could calculate the option prices easily by using the method of Hull and White (1987) and use the BS equation conditional on the volatility path. This would make the simulation faster as then we would only need to simulate the volatility path, calculate average volatility and use that as input in the BS. Then we could just simulate N volatility paths and BS prices and calculate the option price as the average of those N prices. Whether the correlation between returns and volatility changes matters has been one of the most central topics in empirical studies comparing different option pricing models. And indeed, allowing for the correlation would seem to be reasonable, based on the studies of Bakshi et al. (1997, 2000) and Bates (1991, 2000). Their estimates for the correlation efficient differ in the range of [-0.76, -0.25], depending on the time period of data, estimation method and time-to-maturity as well as moneyness of the option. Nandi (1998) focused in his study exclusively on this topic and found that non-zero correlation in a stochastic volatility model leads to significant improvements in pricing and hedging options. He even states that a BS model with daily-adjusted volatility outperforms a stochastic volatility model with zero correlation, but the non-zero correlation version yields the best results. Based on these findings we have to conclude that non-zero correlation really seems to be a vital factor affecting the option price and can not be approximated by zero.

There are several theoretical explanations for the negative correlation, but the two most important are the leverage effect and volatility feedback effect. Leverage effect was discovered by Black (1976), who found that volatility is typically higher after a fall than after a rise in prices, causing negative correlation. Black reasoned that this could be due to the increase in leverage that occurs when the market value of a firm declines. Although the leverage effect is commonly considered as an important factor explaining the correlation, it may not be the only factor, if even a correct one. This has been discussed by Figlewski and Wang (2000), who suggest that the 'leverage effect' is actually a 'down market effect' that may have little direct connection to firm leverage.

Volatility feedback in turn has been discussed by e.g. Campbell and Hentschel (1992). The theory states that a large piece of good news, which increases volatility, tends to be followed by more good news, which again increases volatility and thus raises the required rate of return and lowers the stock price, making the positive effect of the news more moderate. Now when bad news arrive, again the stock price falls because of volatility increase, but now the volatility effect amplifies the negative impact of the news. Therefore large negative changes are more common than large positive ones, and this causes negative correlation between the returns and volatility. Andersen et al. (2001) also found more support for the volatility feedback effect than for the leverage effect.

The first attempt of deriving a closed-form solution formula for the option price when the volatility follows a diffusion process, was the paper by Stein and Stein (1991). The shortcoming of their model was that it did not allow for correlation. They also used an arithmetic OU (or AR1) process for the volatility (and not for its logarithm), which made it possible for the volatility to be negative. A more complete solution, i.e. one without these drawbacks and that also allowed stochastic interest rates, was presented by Heston (1993). The solution technique he presented was based on characteristic functions and Fourier inversion methods. The technique was later applied by Bates (1996a), whose model allowed stochastic volatility and jumps, but not stochastic interest rates, and Scott (1997), whose model allows for stochastic volatility and interest rates, as well as jumps in the price process. Also Duffie et al. (2000) have discussed the transform analysis for a fairly general class of affine jump-diffusion models. However, to be able to include also t-distributed innovation terms in the models, we will not be using the models of Heston, Bates or Scott, but will calculate the option prices by Monte Carlo simulation. Therefore we will not go through these relatively complex models here in more detail.

Usually in empirical applications of the SV model it is assumed that the conditional distribution of returns, given the volatility process, is normal. In other words, the standard assumption is that $dW_{s,t}$ is normally distributed. This assumption is of course implicit when assuming that W is a standard Brownian motion. However, as for example Liesenfeld and Jung (2000) showed, if a more heavy-tailed conditional distribution, such as the Student's t-distribution, is used, it seems that more accurate option prices can be calculated. With this assumption we might be able to further improve the performance of SV models, as with these processes the distribution of returns becomes more leptokurtic. Of course this alternative approach will also affect the parameter estimates. This assumption, which has not been included in the empirical comparisons by e.g. Bakshi et al. (1997, 2000), can easily be studied when pricing the option through Monte Carlo simulation. If using for instance the model of Scott (1997), comparisons between normal and t-distributed error terms would not be possible.

These kinds of models have been previously left with relatively little attention, but Liesenfeld and Jung (2000) have obtained some very promising results. Exploiting the fat tails of the t-distribution we might be able to avoid including jumps in the processes and thus simplify the estimation procedure considerably. In this thesis the model with t-distributed innovations in the asset price process (from now on SVT) is defined as in equation (3.1), only $dW_{t,y}$ is t-distributed, so W_t is no longer a standard Brownian motion. The volatility process remains as in (3.2).

3.1.2 Adding the Jump Component

The findings of Bakshi et al. (1997) and Bates (1996a, 2000) show that even though they find the stochastic volatility to be the most significant factor in explaining the pricing biases of the BS model, allowing the asset price to jump does also have a significant effect on the pricing performance of the model. More recently, even stronger conclusions have been drawn by Andersen et al. (2002) and Pan (2002) who find that, at least for equity index returns, allowing discrete jumps is just as essential as stochastic volatility with negative correlation between return and volatility innovations.

Bakshi et al. (1997) and Bates (2000) also find that even after allowing jumps in the price process, implausible parameters of the volatility process can be needed for the model to be consistent with observed data. Therefore they speculate whether jumps should be included also in the volatility process. Also Pan (2002) finds evidence of possible jumps in the volatility process. Pan (2002, pp. 32-33) argues that because the sample estimates of the third and fourth moments of volatility are found to be positive and significantly different from zero, jumps or at least fatter-tailed innovations in the volatility process are implied. Her overall findings, however, are mixed. Nevertheless, these empirical findings give reason for looking into the class of bivariate jump-diffusion models more closely.

Including jumps in both processes is possible in a class of models presented by Duffie et al. (2000) and used by Eraker, Johannes and Polson (2002). We write the model as

$$d\begin{pmatrix} \log S_t \\ V_t \end{pmatrix} = \begin{pmatrix} r - \frac{1}{2}V_t - \lambda\mu_s \\ \kappa(\theta - V_t) \end{pmatrix} dt + \sqrt{V_t}\begin{pmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{1-\rho^2}\sigma_v \end{pmatrix} dW_t + k dq_t \qquad (3.4)$$

where $W_t = [W_{t,s}, W_{t,v}]^T$ is a standard Brownian motion in $\Re^2$, $q_t = [q_{t,s}, q_{t,v}]^T$ is a pair of (potentially equal) Poisson processes with constant arrival intensities λ_s and λ_v (i.e. $Prob(dq_t = 1) = \lambda$), and $k = [k_s, k_v]^T$ is a vector of jump sizes where k_s, and k_v are the jumps in (log)returns and volatility, respectively, and μ_s and μ_v are the expected jump sizes. ρ is the correlation efficient between $W_{t,s}$, and $W_{t,v}$. The parameters κ, θ and σ_v are defined similarly as in (3.2).

From this model we get the following special cases to be examined empirically in this thesis (we adopt the abbreviations from Eraker et al (2002)):

- Stochastic Volatility (SV). Pure diffusion model without jumps in either process ($\lambda_s = \lambda_v = 0$). Cf. (3.1) and (3.2).
- Stochastic Volatility with Jumps in returns (SVJ). The SVJ model has Poisson jump arrivals in the return process with normally distributed jump sizes $k_s \sim N(\mu_s, \sigma_s^2)$. A combination of (2.12), (3.1) and (3.2).

- Stochastic Volatility with Independent Jumps in the return and volatility processes (SVIJ). Both processes have independent jump arrivals in them, with arrival rates λ_s and λ_v, and independent jump sizes $k_s \sim N(\mu_s, \sigma_s^2)$ and $k_v \sim \exp(\mu_v)$.
- Stochastic Volatility with Correlated Jumps in returns and volatility (SVCJ). The SVCJ model has contemporaneous Poisson jump arrivals in returns and volatility, $q_{t,s} = q_{t,v}$ with arrival rate $\lambda_s = \lambda_v$ and correlated sizes $k_v \sim \exp(\mu_v)$ and $k_s | k_v \sim N(\mu_s + \rho_J k_v, \sigma_s^2)$.

There are also alternative specifications of the volatility model. Some authors, for example Chernov et al. (2002), argue that the ideal model for volatility would not be a jump-diffusion process but a two-factor model, with one factor for controlling the volatility persistence and another one the fat tails. If this kind of an approach is selected, then the best model, according to Chernov et al.(2002, p. 19), turns out to be "a two-factor logarithmic SV specification with possible feedback, the latter causing volatility of volatility to increase".

3.2 Autoregressive Conditional Heteroskedasticity

The literature concerning autoregressive conditional heteroskedasticity (from now on ARCH) was initiated by Engle's paper in 1982. Since then, there has been a massive amount of research contributing to this field, literally hundreds of papers studying ARCH-type models or related issues have been published during the last two decades. The idea behind all the ARCH models is that the volatility is not only time-variant but also autoregressive. This property is used to forecast volatility at some time, given the historical data. Naturally we can and will not go through all of the ARCH models ever presented, but will focus on a few most relevant – from the option-pricing point of view. First we will present briefly the most important ARCH-type models in this context in section 3.2.1. These models include the original ARCH of Engle (1982), the Generalized ARCH (GARCH) of Bollerslev (1986), the Exponential GARCH (EGARCH) of Nelson (1991) and the Nonlinear Asymmetric GARCH (NGARCH) of Engle & Ng (1993). These are the models most often applied to option pricing. For an overview of other ARCH-related work over the years, see Bollerslev et al. (1992), Taylor (1994) or Ghysels et al. (1996). In Section 3.2.2 we will discuss these types of models more in relation to option-pricing.. However, in this thesis ARCH-models will not be studied empirically for reasons discussed later on.

3.2.1 Some ARCH Models

The original ARCH was presented by Engle (1982). In this model the idea is to explicitly recognize the difference between conditional and unconditional variance and to model the conditional one. In other words, we take into account the information of past variances we have and model future variance conditional on this information, i.e. base our expectations on historical data.

Following Engle, an ARCH(q) process is a discrete-time stochastic process $\{y_t\}$ of the form

$$\begin{aligned} y_t &= z_t \sqrt{h_t}, \\ h_t &= \omega + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 \end{aligned} \tag{3.5}$$

where z_t i.i.d. with mean zero and variance of one (conditional on the information set at time t-1, denoted ϕ_{t-1}), q is the order of the ARCH(q) process and $\alpha=(\omega,\alpha_1,\ldots,\alpha_\pi)$ is a vector of unknown parameters. Often it is also assumed that z_t is normally distributed.

A more general class of processes, Generalized ARCH (GARCH), was introduced by Bollerslev (1986). In his paper Bollerslev (p. 308) compares the extension of ARCH into GARCH with the extension of the standard AR-process for time series to the more general ARMA-process. He also argues that when using the ARCH model, a long lag (a large q) in the conditional variance equation is often needed in empirical applications. In his GARCH model he allows lagged conditional variances to enter the equation in addition to past sample variances, which were the only variables specifying the ARCH process. This way a kind of a learning mechanism is included in the model. Otherwise the assumptions about the process $\{y_t\}$ remain the same.

The GARCH(p,q) process is given by:

$$y_t = z_t\sqrt{h_t},$$
$$h_t = \omega + \sum_{i=1}^{q}\alpha_i y_{t-i}^2 + \sum_{j=i}^{p}\beta_j h_{t-j} \tag{3.6}$$

where

$$z_t|\phi_{t-1}\ i.i.d.(0,1)$$
$$p \geq 0,\quad q > 0$$
$$\omega > 0,\quad \alpha_i \geq 0,\quad i = 1,...,q$$
$$\beta_j \geq 0,\quad j = 1,...,p$$

For p = 0 the process reduces to the ARCH(q) process, and for p = q = 0 y_t is just white noise.

However, as Nelson (1991, pp. 348-350) points out, there are some drawbacks with linear GARCH models. When pricing options, the most important drawback is that while the GARCH models do account for volatility clustering in a very effective way, they do not account for another widely recognized phenomenon, namely the (negative) correlation between stock returns and changes in return volatility.[9] That is caused by the assumption in GARCH models that only the magnitude and not the sign of excess returns has an effect on the variance. Another limitation of GARCH arises from the non-negativity constraints of ω, α_i:s and β_j:s in equation (3.6). They are initially included in the model to ensure that h_t remains nonnegative for all t with probability one. However, the constraints also imply that increasing y_t^2 in any period t increases h_{t+k} for all $k \geq 1$, ruling out a process in which h_{t+k} can behave so that it grows and diminishes randomly at different periods. The non-negativity constraints can also make the estimation of GARCH parameters more difficult. The third drawback of GARCH

---

[9] This aspect was discussed in more detail in section 3.1 as it is also important with stochastic volatility models.

models concerns the persistence of shocks to conditional variance. If there is a shock at some time t, how long will it have an effect on future GARCH estimates? To overcome these drawbacks Nelson presented an alternative model, the EGARCH. However, one should not forget that in spite of these limitations the GARCH model is, as previously mentioned, still probably the most widely used framework for modeling conditional variances.

The Exponential GARCH (EGARCH), presented by Nelson (1991), is a nonlinear ARCH model that is also very popular. In EGARCH, the non-negativity of variance is ensured by constructing a model for the logarithm of variance, as opposed to the variance itself. This gives the EGARCH(1,1)[10] model the following form:

$$\log(h_t) = \omega + \alpha\left[\upsilon\frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma\left(\frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}}\right)\right] + \beta\log(h_{t-1}), \tag{3.7}$$

where

$$\varepsilon_t|\phi_{t-1} \sim N(0, h_t)\ \textit{and thus}\ \frac{\varepsilon_t|\phi_{t-1}}{\sqrt{h_t}} = z_t|\phi_{t-1} \sim N(0,1)$$

By construction, the formula multiplied by α, say $g(z_t)$, is a zero-mean, i.i.d. random sequence. The components of $g(z_t)$, υz_t and $\gamma(|z_t|-E|z_t|)$, both have mean zero. If the distribution of z_t is symmetric (e.g. $z_t \sim N(0,1)$), the components are orthogonal and thus uncorrelated, though of course not independent of each other. Furthermore, when $z_t > 0$, $g(z_t)$ is linear with slope $\upsilon+\gamma$ and when $z_t \leq 0$ it is linear with slope $\upsilon-\gamma$. Hence, $g(z_t)$ allows the conditional variance process $\{h_t\}$ to respond asymmetrically to falls and rises of the stock price. (Nelson, 1991, p. 351) In his paper Nelson (1991, pp. 351-353) also argues why the EGARCH model meets the objections raised to the GARCH model.

[10] Generalization of this equation into EGARCH(p,q) is straightforward, as with any other GARCH model. From now on we will, for simplicity, use p=q=1 for EGARCH and NGARCH models, as they are the ones usually applied in the option-pricing literature.

The Nonlinear Asymmetric GARCH (NGARCH), first presented by Engle & Ng (1993), is very similar to the linear GARCH of Bollerslev (1986). It has only been modified in such a way that it allows for the negative correlation between returns and volatility innovations. This is why this model is often used instead of the basic linear GARCH. Also Duan (1996b, p. 55) recommends using the NGARCH-specification when pricing options in GARCH framework. The NGARCH(1,1) models the variance as follows:

$$h_t = \omega + \alpha h_t \left(z_{t-1} - c\right)^2 + \beta h_{t-1} \tag{3.8}$$

where

$$\omega > 0, \alpha, \beta \geq 0, z_t \left| \phi_{t-1} \sim N(0,1) \right.$$

$$\textit{If } c > 0, \textit{ it captures the negative correlation.}$$

3.2.2 ARCH Models in Option Pricing

Initially, when the GARCH processes were developed, the aim was not to be able to price options more efficiently, but the reason for these models was somewhere else, like for example modeling macro-level economic factors such as inflation. Maybe that is why the GARCH models have not been studied as extensively in the option pricing literature as diffusion processes with stochastic volatility. Another reason could be that the theoretical foundations of option pricing are in continuous-time finance and in stochastic differential equations. In this framework the stochastic volatility models fit easily, whereas the GARCH models require a slightly different point of view. Rather than a mathematical model based on economic theory about the asset price process and the driving factors behind it, the GARCH model is a powerful econometric tool, based on empirically discovered statistical properties of a process. One thing making GARCH modeling more attractive in comparison to diffusion modeling is that it is easier to estimate. This is caused by the fact that in bivariate diffusion models the variance (or volatility) process is unobservable, whereas in the GARCH setting parameters can be directly estimated from historical data.

One of the first published studies linking the GARCH literature with option pricing theory was Engle & Mustafa (1992). The first paper to provide a rigorous theoretical foundation for using GARCH models in option pricing was not published until 1995 by Duan. It is to be noted that Duan achieved his results applying equilibrium-type arguments, but Kallsen and Taqqu (1998) have later shown that a similar pricing formula can also be derived using the no-arbitrage condition. Other extensions to Duan's work have been provided e.g. by Ritchken and Trevor (1999), Heston and Nandi (2000), as well as Duan himself (1996a, 1996b).

In order to be able to price options under a GARCH process, a generalized version of the risk neutral measure is needed. This was derived by Duan (1995), who named it the locally risk-neutral valuation relationship (LRNVR). A detailed definition of the LRNVR can be found in Duan's original paper. (1995, Definition 2.1, pp.15-16) Comparing LRNVR with the traditional notion of risk-neutral valuation, the LRNVR is weaker in the sense that it is insufficient for totally eliminating the preference parameters. It is nevertheless strong enough to reduce the preference consideration into a single parameter λ, which can be interpreted as the unit risk premium. The LRNVR does not, however, locally depend on preferences. This is implied by the properties that the conditional variances under the two measures P and Q are equal and that the conditional expected profit can be replaced by the risk-free rate. These properties are similar to those of the traditional risk-neutral measure. Duan (1995) also proves that LRNVR is valid under similar assumptions about preferences and distributions as the assumptions under which the risk-neutral valuation, as discussed by Rubinstein (1976) and Brennan (1979), holds.

The implication of the LRNVR is presented below (see Theorem 2.2 of Duan 1995, p. 17). The general theory of GARCH option pricing applies to all GARCH models[11], but here we will only demonstrate on NGARCH(1,1).

[11] See e.g. Duan (1995 and 1996b), as well as Schmitt (1996).

Under LRNVR, when the underlying process is NGARCH(1,1),

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{1}{2}h_t + \sqrt{h_t}\xi_t, \tag{3.9}$$

where

$$\xi_t|\phi_{t-1} \sim N(0,1)$$

and

$$h_t = \omega + \alpha h_t\left(\xi_{t-1} - c - \lambda\right)^2 + \beta h_t \tag{3.10}$$

The proof of (3.9) and (3.10) can be found in Duan (1995, p. 27). Note that these results depend on the normality assumption and therefore error terms with more fat-tailed distributions, such as the t-distribution, can not be included in the GARCH framework. These results also imply constant interest rates over time. According to Duan (1995, p. 16) it is possible to develop a model with stochastic interest rates also in the GARCH framework, but the resulting model would be considerably more complicated. In addition to NGARCH perhaps the most often applied GARCH model with respect to option pricing is the EGARCH of Nelson (1991). Deriving the locally risk-neutral EGARCH process is explicitly presented for example in Schmitt (1996).

As discussed, there is an important fundamental difference between the ARCH models and the diffusion models. The ARCH-models are discrete-time models, initially a powerful econometric tool based on empirical findings, whereas the SV-models are continuous-time models that have their foundations in continuous-time theory. Still these models are not always so different, as for example Taylor (1994) discusses, but rather some ARCH models have their continuous-time limits in SV models. This convergence has been studied in more detail by Nelson (1990), Duan (1996a) and Ritchken and Trevor (1999), among others. This finding can sometimes be very important, as it can make the usage and estimation of both models easier under certain circumstances.

The similarity of the results from the two model classes is also the main reason for us not examining the ARCH-type models in the empirical part of this thesis. The results should theoretically be very similar regardless of whether a GARCH- or a SV-process is specified for the volatility. Also, in a (G)ARCH setting it is much more difficult to include jumps in the price process, let alone the volatility process. Even though a model with both the GARCH-effect and jumps has been introduced by Drost, Nijman and Werker (1998), applying this model for option pricing might carry more down- than upsides with it. More or less the same results can be achieved more easily with SV models than with GARCH models, which are discrete-time models by nature. An alternative way to model an equivalent of jump diffusion processes in a discrete-time setting was presented by Amin (1993), who incorporated jumps in the binomial tree approach of Cox et al. (1979). This method will not, however, be discussed more extensively in this thesis.

3.3 Constant Elasticity of Variance and the Implied Tree Approach

The constant elasticity of variance (CEV) option pricing model was first presented by Cox and Ross (1976). The model can be written as

$$dS/S = \mu dt + \sigma S^{\rho-1} dW \qquad (3.11)$$

which is actually a more general form of GBM, because when $\rho = 1$, the CEV model reduces into GBM. In the original paper by Cox and Ross (1976) the analysis was constrained only to special cases $\rho = 0$ and $\rho = ½$.

There are some properties in this model that made it initially attractive, but also some drawbacks that seem to outweigh the positive aspects. Here we only mention a couple of the most important issues, both positive and negative. A more detailed discussion can be found for example in Bates (1996b, pp. 586-587). What is the biggest attraction of the CEV model is that it is consistent with Black's (1976) observation about the negatively correlated returns and volatility changes. Therefore it was initially considered to have potential to explain some biases of the BS model. The most important drawback is that the variance is modeled as a deterministic and monotonic function of the underlying nominal asset price. Then, given that asset prices typically have unit roots and non-zero drift, the CEV model with $\rho \neq 1$ implies that the variance approaches either infinity or zero in the long run. Also the fact that the CEV model is consistent with bankruptcy, which may be a good property when pricing a stock option, is probably not desirable when dealing with index or currency options.

The more recent models of implied (binomial) trees (Rubinstein 1994, Derman & Kani 1994), can actually be viewed as generalizations of the CEV models (Bates 1996b, p. 586), as they also model instantaneous volatility as a flexible but deterministic function of the asset price and time. The idea in the implied tree models is that the observed smile – the prices of European options of all maturities and strike prices – are used as input in a backwards recursive method for inferring the 'implied' risk-neutral probabilities. These probabilities are then used to infer a unique, fully specified and recombining binomial tree, which can in turn be used to price other, also American-style and exotic, options.

Unfortunately there has also been some criticism of the implied tree models. For example, Bates (1996b, p. 587) claims that these models suffer from the same problem as CEV models discussed above, namely that the variance approaches either zero or infinity in the long run. Therefore both types of models require repeated parameter recalibration, which in turn indicates fundamental misspecification. Dumas et al. (1998) examined the predictive and hedging performance of the implied tree models, and found that in the end they do not perform any better than an 'ad-hoc' BS model that merely smoothes the BS implied volatilities across exercise prices and maturities. Also Hagan et al. (2002) studied the hedging performance of implied tree models and found that they can perform even worse than the BS model. For these reasons the implied tree (or CEV) models will not be studied in the empirical part of this thesis.

Nevertheless, despite all the shortcomings of these "calibration methods" and our ignoring them, one should not totally dismiss the models. Developing the models to overcome the difficulties is an ongoing process, and among practitioners the models are considered to have potential. One possible way to improve the models' performance is including exotic options in the data set in addition to the traditional European options.

One recent example of this kind of modeling is presented by Johnson and Lee (2003), who claim to achieve an exact fit to all the chosen market prices, yet reducing the danger of overfitting. However, the fundamental theoretical problems discussed above still remain underneath.

3.4 Summary of the Models

In this section we will briefly summarize the models studied in this thesis. We will not go through the theoretical details again, those can be found in sections 2.1-2.4 and 3.1-3.3.

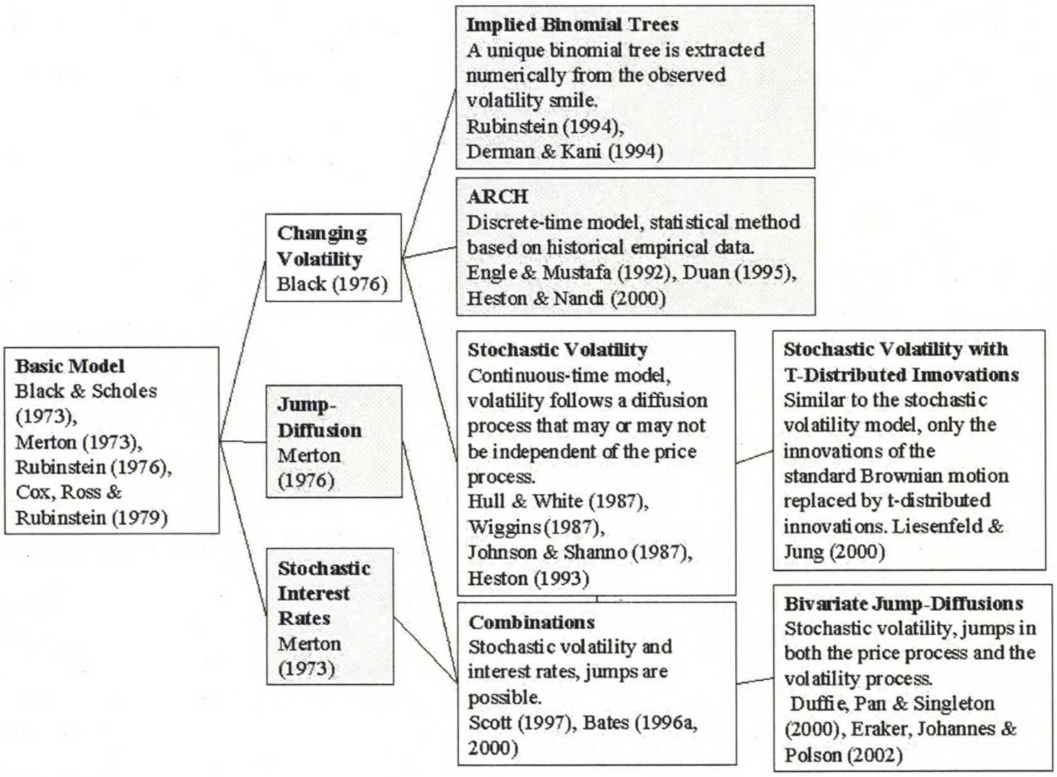


Figure 2, Evolution of Option-Pricing Models

In the figure are depicted the evolution of the most important option-pricing models since the early 1970's and the most important papers related to the respective models. The models in white boxes are those that are empirically studied in this thesis whereas the models in grey boxes are only discussed but not empirically studied.

In the basic models, most notably the BS (see section 2.1), the volatility and interest rates are assumed constant over the life of the option and the price process is assumed continuous. These are the main assumptions of the classic models that are often criticized. At least one of these assumptions is also relaxed in each of the models studied in this thesis, only the constant interest rates-assumption is respected throughout the empirical part of the thesis.

There are several methods to model time-variant volatility and we study empirically only some of these. The other methods can also be argued to have their strengths, but for our purposes the chosen methods suit best. We cover models from the basic stochastic volatility to the models including jumps in the volatility. We will not study the ARCH-models (see section 3.2) or local volatility (often also referred to as the implied tree) models (see section 3.3) more closely. The models discussed and studied in the thesis can be seen also in figure 2 above. The papers mentioned in the figure form by no means an exhaustive list of relevant references, but are only a few examples of the most important ones.

4 Data and Methodology

This chapter describes the data used in the empirical study. Also the methods used for estimation of the model parameters and the comparison of the models will be presented. The pricing models will be compared in terms of both the static and dynamic performance, i.e. pricing and hedging ability, respectively.

4.1 Data

In the empirical part of this study European call options on two different underlying assets, the DJ Euro Stoxx 50 index and the stock of Nokia Corporation, will be studied.

The first asset is the Dow Jones Euro Stoxx 50 index, which consists of fifty blue-chip companies from the Eurozone (for more detailed information about the index, see STOXX Ltd., 2003). The reason for using exactly this index out of many is that in Eurex derivatives market, the options on this index are among the most actively traded (Eurex, 2003). This will lead us to believe that the market for options on this index is the most efficient, yielding the most accurate market quotes for the options. Therefore it can be thought of as the best market in Europe to compare the option-pricing models.

The other underlying asset considered in this thesis is the stock of the Finnish telecom company Nokia Corporation (see Nokia, 2003). This company's stock was chosen as a representative asset for this study because, similarly to the DJ Euro Stoxx 50 index, during recent history the options on this stock have been among the most actively traded options in the Eurex market. It is interesting to see whether the pricing and hedging performances of the pricing models differ when studying options on an individual company's stock, as opposed to index options. Also the price performance of an individual stock, especially as the company operates in telecom industry, is likely to be much more volatile and therefore more might be demanded of the option-pricing model. Furthermore, different features of a model might have to be emphasized, depending on whether an option on an index or a stock is under scrutiny.

The option price data for this study was provided by the European derivative exchange Eurex, which is a part of the Deutsche Börse (see Eurex, 2003, for more information) and is located in Frankfurt am Main, Germany. We had available option price data for the time period of two years, from July 3, 2000 to June 28, 2002. Index and stock price data was obtained from the Datastream database, but as the dividend-adjusted data was not available, we followed Duan (1996b) and used the theoretical put-call parity (see section 2.1) to obtain the dividend-adjusted price data from the option prices. The performance of DJ Euro Stoxx 50 index and Nokia stock, both the parity-implied and the actual (not dividend-adjusted), are depicted in figures 3 and 4. It can be seen that the effect of the dividend-adjustment is relatively small on this scale. It can, however, still be significant in pricing and hedging options as the models can be very sensitive to even small changes in the underlying asset prices.

The maturities of the options considered were between 1 and 364 days, as the data implied that the trading of options with maturities longer than one year is usually very thin. Because of the large number of options traded on each asset each day, we were able to leave out the options that were not actively traded. This was done by including only option price quotes that were based on at least five hundred daily trades. This way we can assume that the quote truly represents the correct valuation of efficient markets. These actively traded options were studied in both comparisons, i.e. the one based on the pricing ability and the one on hedging ability of a model. However, in order to have enough data for computing the hedging errors, the next day's prices of the options, i.e. the price after the hedging period Δt (see sections 2.6 and 3.4), are allowed to be also among the less traded options.

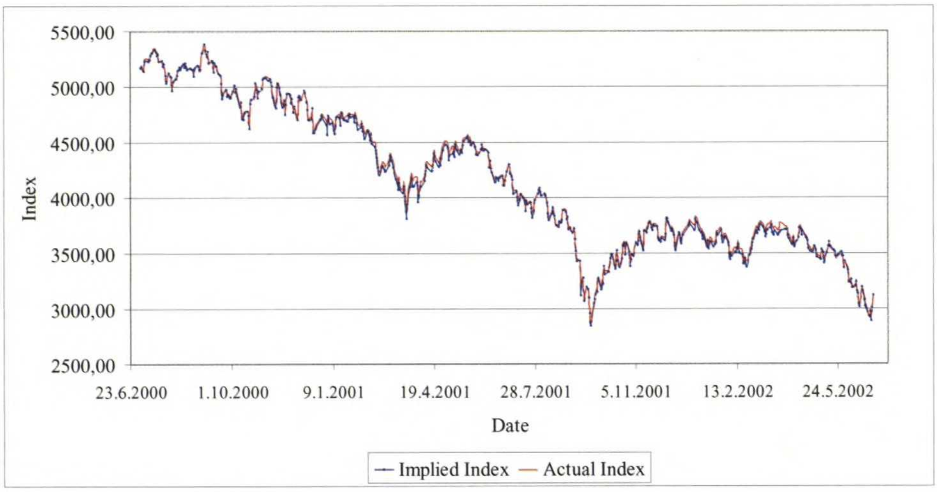


Figure 3, Performance of the DJ Euro Stoxx 50 Index

In the figure are depicted the index level implied by the put-call parity and the actual one obtained from Datastream. In the actual index data no dividend adjustment is made. The index data implied by the put-call parity is automatically dividend-adjusted. The time period in the figure covers both our in-sample estimation period and the out-of-sample control period.

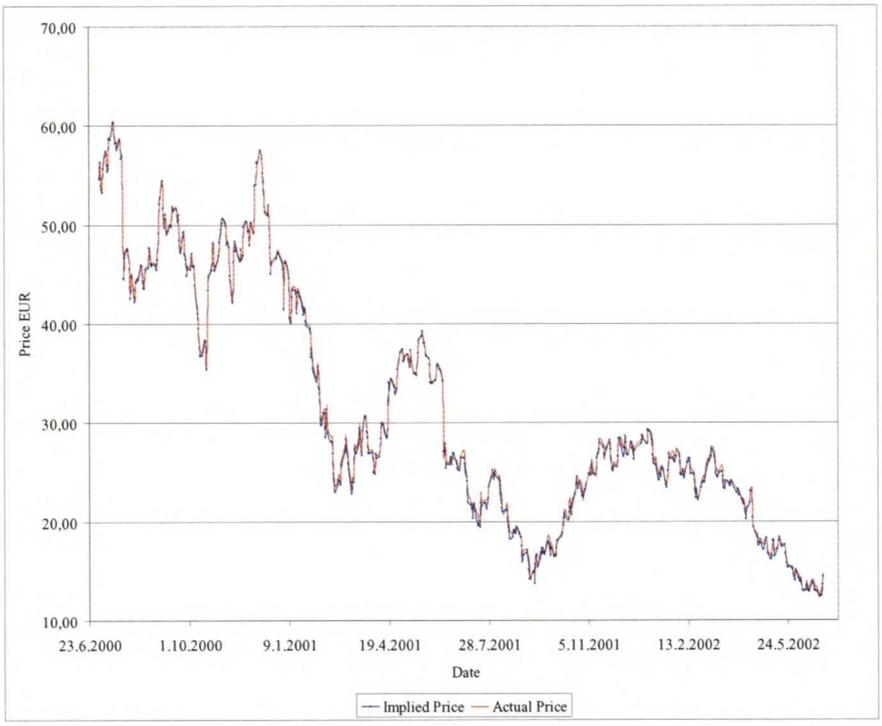


Figure 4, Performance of the Nokia Stock Price

In the figure are depicted the Nokia stock price implied by the put-call parity and the actual one obtained from Datastream. In the actual price data no dividend adjustment is made. The price data implied by the put-call parity is automatically dividend-adjusted. The time period in the figure covers both our in-sample estimation period and the out-of-sample control period.

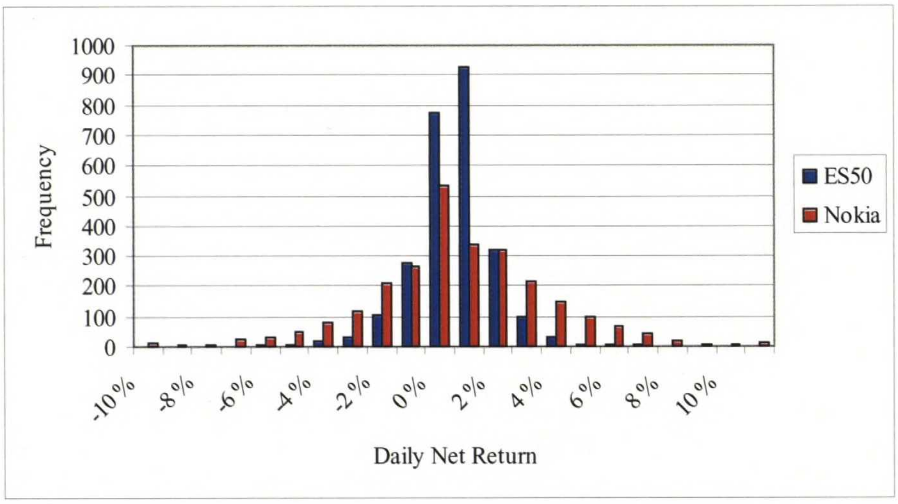


Figure 5, Historical Daily Net Returns of the DJ Euro Stoxx 50 Index and Nokia Stock.

In the figure are depicted the daily net return distributions of the index and of the stock. Dividends are included but taxes are ignored. There was a total of 2610 observations during the ten-year period March 29, 1993 to March 28, 2003. ES50 stands for the DJ Euro Stoxx 50 index.

In figure 5 above are depicted the daily net returns of both the DJ Euro Stoxx 50 index and of Nokia stock during a ten-year period March 29, 1993 to March 28, 2003. These represent the long-term distributions of daily returns, though of course the distributions during different time periods can be very different. The returns were obtained from Datastream database and the dividends are assumed to be reinvested in the same asset. Taxes have been ignored. The main statistical properties of the distributions are stated in table 1 below, along with the results of the Shapiro-Wilk[12] test for normality.The null hypothesis of normal return distributions was overwhelmingly rejected as the p-values were, for both distributions, smaller than $2.2*10^{-16}$. This result of course suggests that the BS assumption of lognormal prices is incorrect.

---

[12] The Shapiro-Wilk test calculates a W statistic that tests whether a random sample, $x_1, x_2, ..., x_n$ comes from (specifically) a normal distribution. The W-statistic is calculated as $W = \left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2 / \sum_{i=1}^{n} (x_i - \bar{x})^2$, where the x(i) are the ordered sample values (x(1) is the smallest) and the a_i are constants generated from the means, variances and covariances of the order statistics of a sample of size n from a normal distribution. The smaller the W-statistic, the more evidence of departure from normality. See also R Homepage (2003) and the references therein.

Table 1, Properties of the Historical Daily Net Return Distributions

In the table are depicted the first four moments, i.e. the mean, variance, kurtosis and skewness, of the returns distributions of for DJ Euro Stoxx 50 index (ES50) and Nokia stock The Shapiro-Wilk test for normality has been conducted and for it the W-statistics are reported. *** represents statistical significance on the 0.01% level.

	ES50	Nokia
Mean	0.03 %	0.15 %
Variance	0.00019	0.00105
Kurtosis	3.44	5.30
Skewness	-0.12	-0.40
W-Statistic	0.9488***	0.9568***

Euribor interest rates were used as the proxy for the risk-free interest rate. In figure 6 Euribor term structure curves at different time points during the data period are depicted. Linear interpolation was used to obtain interest rates for maturities other than those quoted. From the figure it can be seen that most of the time the term structure has been relatively flat. When considering the five depicted time points, any significant differences between the one-week and the twelve-month (annualized) interest rates can be observed only on the earliest and the latest time point, i.e. on July 3rd, 2000, and June 28th, 2002. Even then the difference in p.a. interest rates is of the magnitude of half a percentage point.

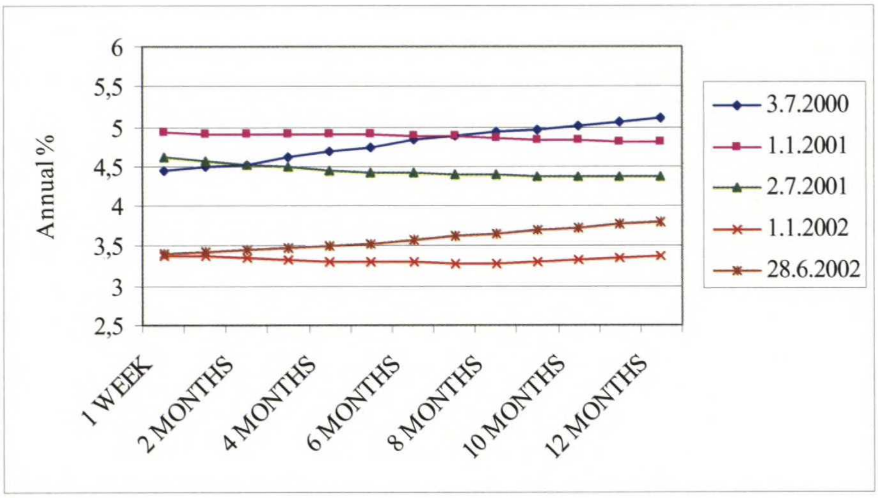


Figure 6, Euribor Curves at Different Points in Time

In the figure are depicted the term structure curves of Euribor interest rates at six different times (day.month.year) during our data period. On the X-axis are depicted the maturities of interest rates and on the Y-axis the annualized interest rates as percentages. Quotes for the interest rates are based on an act/365 day-count.

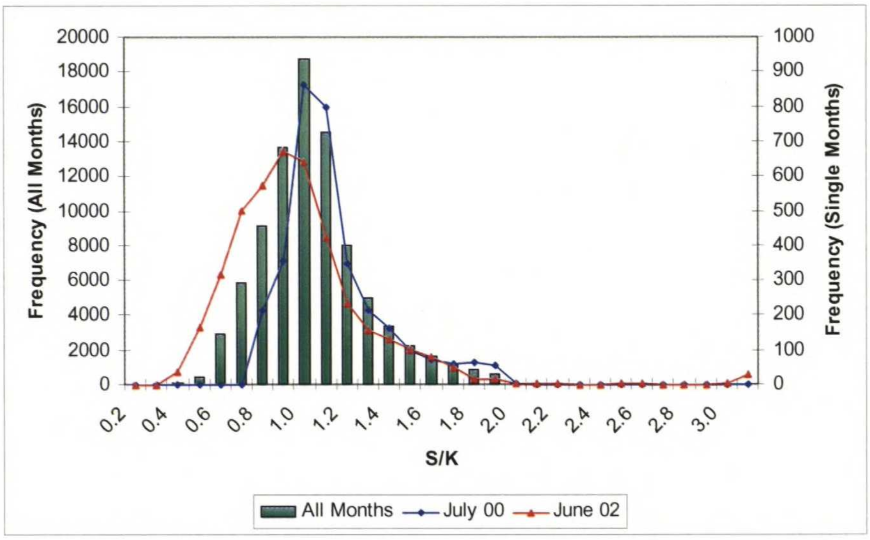


Figure 7, Moneyness Distributions of Options on the DJ Euro Stoxx 50 Index, July 2000 to June 2002

The frequencies of the whole time period (the columns) are depicted on the primary Y-axis on the left-hand side and the frequencies for the single months July 2000 and June 2002 (the lines) on the secondary Y-axis on the right-hand side of the figure. Moneyness S/K is depicted on the x-axis.

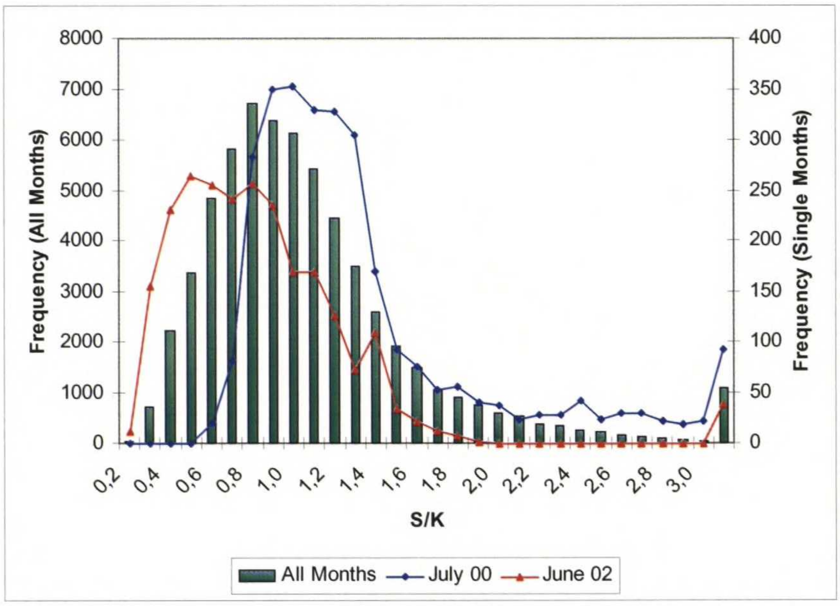


Figure 8, Moneyness Distributions of Options on the Nokia Stock, July 2000 to June 2002

The frequencies of the whole time period (the columns) are depicted on the primary Y-axis on the left-hand side and the frequencies for the single months July 2000 and June 2002 (the lines) on the secondary Y-axis on the right-hand side of the figure. Moneyness S/K is depicted on the x-axis.

More descriptive information of the data is presented in figures 7 and 8, where the moneyness, defined asset price S divided by the strike price K, distributions of options on DJ Euro Stoxx 50 Index and Nokia stock, respectively, are depicted. The columns depict the moneyness distribution of the whole data period July 2000 to June 2002. The blue and red lines depict similar distributions for the first and the last month of the sample period, respectively. From the differences it can be seen that early in the period more options were in-the-money and later the average moneyness has fallen. This can of course be explained by the fall of the stock markets globally, as when the stock prices S fall and strike prices K of open options remain the same, ratios S/K become smaller on average.

Figures 9 and 10 present the implied volatilities, as calculated from the Black-Scholes equation using the Newton method[13], of the index and of Nokia stock, respectively. Only one volatility was calculated for one day, and the option used for that was the one with S/K ratio closest to one (i.e. the option closest to being exactly at-the-money) and time to maturity closest to 30 days. Of course this filtration can cause some error in the volatilities because of the volatility smile phenomenon (see Section 2.5) and because the times to maturity are not equal every day (in a given month there is only one day for exercising European options in Eurex, see Eurex 2003).

For the Euro Stoxx 50 index the mean volatility over the sample period was 27.32% and the median 26.45%. The minimum volatility of the index was 17.96%. The maximum volatility, 63.33%, was reached on 11th of September 2001. Similarly for Nokia stock the mean volatility over the sample period was 67.07%, the median 64.01% and the minimum 48.43%. The maximum volatility of Nokia options, 115.50%, was also reached on 11th of September 2001. In addition to the days following the attacks on September 11th, other volatility peaks took place in the late February 2001 after the annual Nokia shareholders' meeting and 24th of June 2002 after the announcement of the company's growth prospects. (Nokia, 2003) From these

---

[13] Newton method is a method for approximating roots. First you guess an initial value X_0, supposedly close to the root. Then this value is corrected closer to the actual root by $X_0-f(X_0)/f'(X_0)$ to obtain value X_1, which is in turn corrected similarly. This process is iterated until the accuracy is good enough.

figures one can see that our initial assumption of Nokia stock being more volatile than the index seems to be correct.

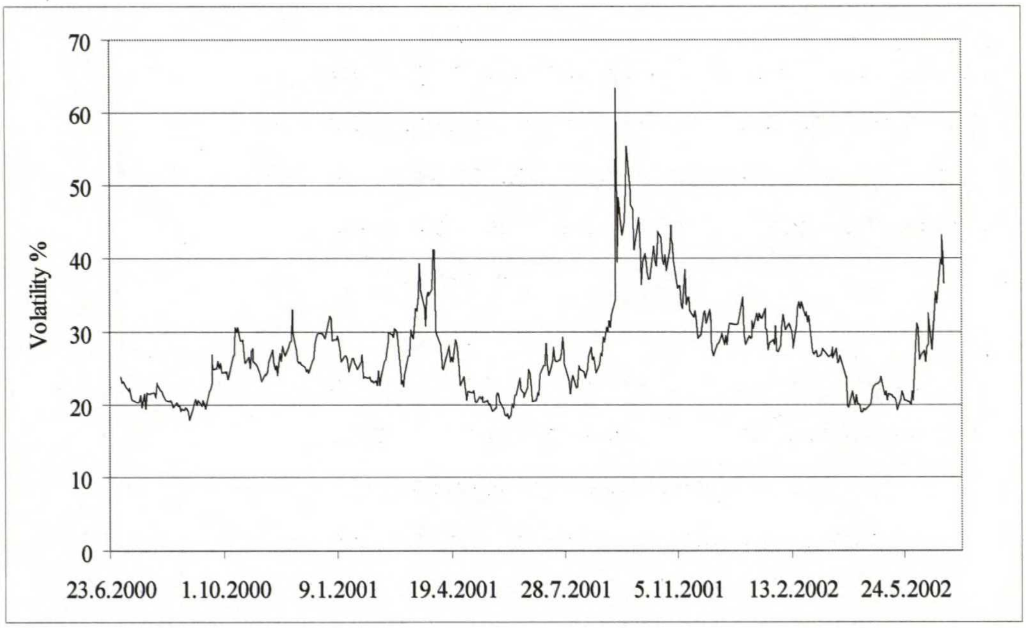


Figure 9, Implied Black-Scholes Volatilities of At-the-Money Options on DJ Euro Stoxx 50 Index

In the figure are depicted BS implied volatilities of one-month ATM options on Euro Stoxx 50 index. Time (day.month.year) is depicted on the X-axis. The first observation is from 3rd of July 2000 and the last from 28th of June 2002. On the Y-axis are the instantaneous volatilities as percentages.

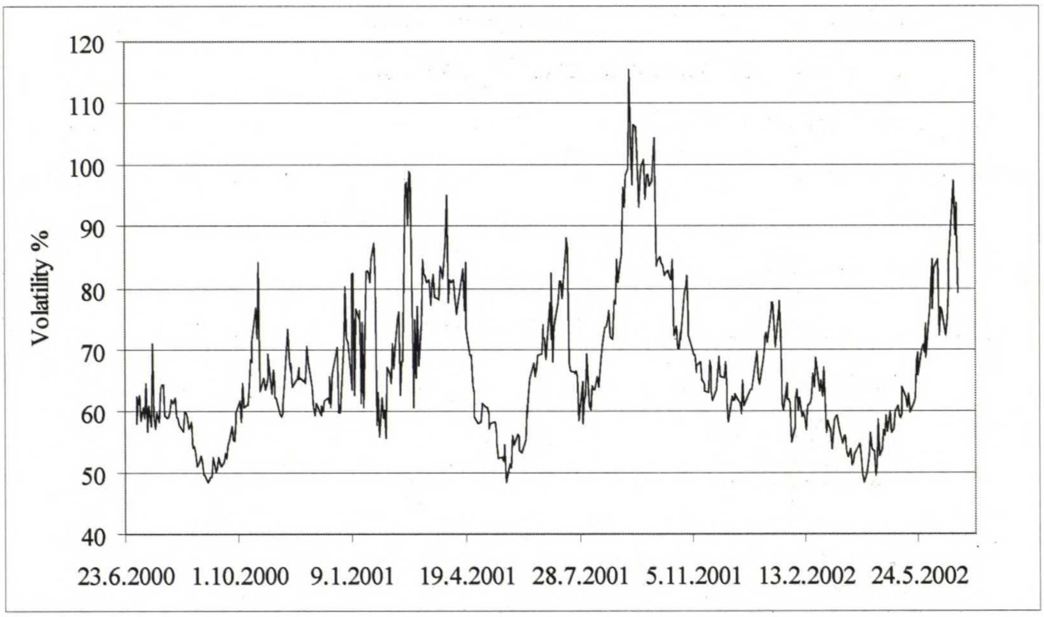


Figure 10, Implied Black-Scholes Volatilities of At-the-Money Options on Nokia Stock

In the figure are depicted BS implied volatilities of one-month ATM options on Nokia stock. Time (day.month.year) is depicted on the X-axis. The first observation is from 3rd of July 2000 and the last from 28th of June 2002. On the Y-axis are the instantaneous volatilities as percentages.

The implied volatility time series also seem to be significantly autocorrelated. This can be seen in figure 11, which depicts the autocorrelation functions (ACF) for implied volatilities of options on (i) DJ Euro Stoxx index and (ii) Nokia stock. In both series the autocorrelation coefficient seems to decrease more or less linearly as the lag increases. In the volatility series calculated from the index options the ACF seems to disappear around the 50-day lag, and in the series calculated from Nokia options the ACF seems to disappear around the 40-day lag. The existence of a strong serial correlation in both of the volatility time series is confirmed by the Ljung-Box test, which tests the joint significance of the ACFs for lags 1 to 50 for the index volatilities and for lags 1 to 40 for the Nokia volatilities. In the Ljung-Box test the null hypothesis states that all autocorrelation coefficients equal zero. For more information about the test, see e.g. Campbell et al. (1997, pp. 47). The test statistic for the index volatilities is 6393.329 and for the Nokia volatilities 4511.477. Both tests imply a p-value smaller than 2.2e-16, so the results are significant at 0.01% level and the null hypothesis is overwhelmingly rejected in both cases. Bakshi et al. (2000, pp. 310-302) carried out the same test with options on the S&P 500 index and their results for the short-term options (in their context maturities less than one year) were almost identical to ours. Their results for longer-maturity options implied even stronger serial correlation. And as Bakshi et al. (2000, p. 302) argue, the smaller autocorrelation of shorter-term options means that the volatility process is not required to be as persistent as in longer-term options. This in turn can be interpreted as evidence against the BS model.

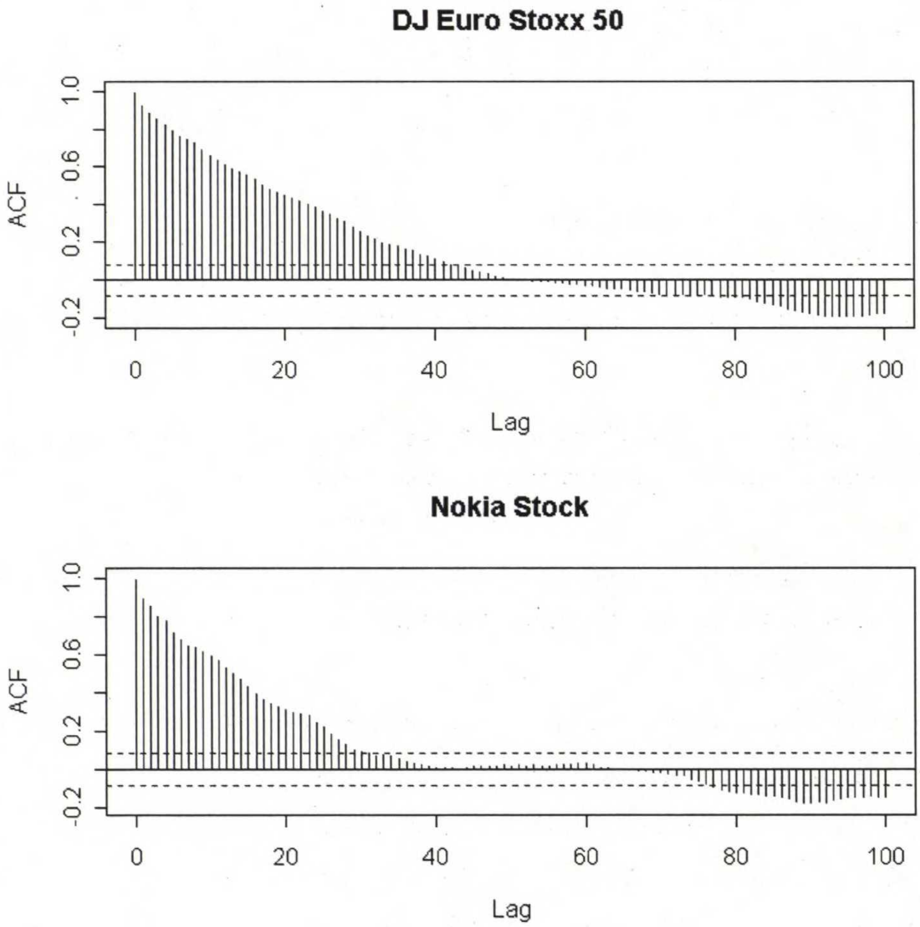


Figure 11, The Autocorrelation Functions of the BS Implied Volatilities

In the figure the autocorrelation functions (ACF) are depicted for BS implied volatility time series as computed from options on (i) DJ Euro Stoxx 50 Index and (ii) Nokia stock. The dotted lines represent the 95% confidence interval and the columns represent the estimated autocorrelation coefficient (values on y-axis) for the respective lags (values on x-axis).

4.2 Estimation of Model Parameters

In this section we will describe the estimation[14] method for the structural parameters in general terms. This method will be applied to all the models studied in this thesis. The results will be reported in section 5.1. For comparison we will also be using the traditional BS model, but for that all the parameters except for the volatility are directly observable from the data. To give BS a fair chance in comparison, we follow Bakshi et al. (1997) and use daily-adjusted volatility as input. This is actually somewhat ad hoc, because the original assumption of Black and Scholes (1973) was that volatility remains constant over time. However, because of the results of Bakshi et al. (1997) as well as of Dumas et al. (1998), this adjustment makes the BS more competitive in comparison with the more complex models and therefore enables comparison on a more equal basis.

In the literature there is a massive amount of different estimation methods used for volatility models. The estimation has, however, proven to be quite cumbersome on discrete-time data, which any possible data naturally always is. There are two reasons for this difficulty. First, because the actual volatility process is unobservable, the maximum likelihood estimation of the parameters is difficult or even impossible in some cases. Consequently, either time series analysis of volatility proxies (such as sample variances) or some kind of method of moments estimation, like the generalized method of moments (GMM), using moments of the unconditional distribution of asset returns, have frequently been applied.

The second problem is that to be able to test the implications of the time series estimates for option prices under SV we need to assess the current level of instantaneous conditional volatility. This brings us to the filtration issue of identifying the volatility level given past information on asset returns. This has been tried e.g. by using an extended Kalman filter, as in Melino and Turnbull (1990). With the ARCH processes the estimation problems would be less severe (about estimation of GARCH models in general, see e.g. Greene, 2000, pp. 796-810 or Campbell et al., 1997,

---

[14] The estimations, as well as all other calculations, were made using the R language. The software and some manuals are available free of charge at the R Homepage.

pp. 479-498), but some problems would still arise because of the differences between the risk-neutral and the true distributions. (Bates, 1996b, pp. 586-592)

A method quite often used in the more recent publications, such as Andersen et al (2002), is the Efficient Method of Moments (EMM). It is an unconditional simulation-based GMM approach that applies to most models and provides model diagnostics and parameter estimates with attractive asymptotic properties. However, estimates for latent variables such as volatility, jump times and jump sizes are not provided because they are integrated out. (Eraker et al., 2002, p. 9) Furthermore, an estimation method called Monte Carlo Markov Chain (MCMC) is sometimes used, for example by Eraker et al (2002). This method also has its attractions, Eraker et al (2002, p. 9) argue that with this method also the latent factors can be estimated and that estimation risk can be controlled for.

In order to overcome the problems discussed above and to find an estimation procedure that is simple and can be applied for all relevant models, a method very similar to those of Bates (2000) and Bakshi et al. (1997) is used to estimate the structural parameters from the option data. This is done by minimizing the sum of squared differences between model prices and observed prices. There are two steps in the estimation procedure:

Step 1. Collect N option prices for $N \geq 1+k$, where k is the number of parameters to be estimated. For each $n = 1,\ldots,N$, let τ_n and K_n be the time-to-expiry and the strike price, respectively, of the nth option. Let $\Phi^m \in \Re^k$ be the parameter vector for model m, such that k is the number of parameters to be estimated. Now denote the observed market price $\hat{C}_n(t,\tau_n,K_n)$, and the model price for model m $C_n^m(t,\tau_n,K_n,\Phi^m)$. Then, for each n and m, define

$$\varepsilon_n^m(\Phi^m) \equiv C(t,\tau_n,K_n,\Phi^m) - \hat{C}(t,\tau_n,K_n) \tag{4.1}$$

Step 2. Find the parameter vector Φ^m to solve

$$SSE \equiv \min_{\Phi^m} \sum_{n=1}^{N} \left[\varepsilon_n^m\left(\Phi^m\right)\right]^2 \tag{4.2}$$

The BS implied volatilities for ATM options (as depicted in figures 9 and 10) are used as proxies for V_0, the initial values for the volatility processes. The advantages of this estimation method are, in addition to simplicity, that not much data is needed and the option-implied parameters can be interpreted as forward-looking, whereas the time-series methods are bound to the historical data. According to Bates (1996b, p. 594), the problem with implicit parameter estimation is that there is no associated statistical theory. This method is comparable with extracting the implied volatility out of the BS model.

A potential problem with this method arises from optimization difficulties. As there can be up to nine parameters to be estimated, i.e. nine free variables in the optimization problem, the optimum value recovered can be quite sensitive to the starting values set to the algorithm. This can in turn lead to a local optimum being obtained. This problem is controlled for by comparing the obtained parameter estimates to the estimates in comparable studies. Furthermore, as the estimates are a result of simulation, some simulation error could be included in the estimates causing inaccuracy. That we were, because of lack of sufficient computing power and a large number of options to be priced, forced to cut down the number of simulation runs to 1500 runs per calculated option price, did not help us much in overcoming this problem but could instead have increased the effect of noise in the results. However, estimation error is always an issue in studies like this, regardless of the estimation method used and it can never be totally eliminated. Still one should keep in mind that some random error could be included the estimates, even if the problem diminishes as the number of simulations increase.

To be able to compare the models, the first step is to try analyzing whether the models are fundamentally misspecified. There are several diagnostics that could be used for this purpose. Eraker et al. (2002) study whether the residual time series from the diffusion model, i.e. the SV model, are normally distributed. If they are, it would imply that the SV specification is correct, because normally distributed error terms are the fundamental assumption behind the model. However, if the residuals are not normally distributed, it would imply the model is misspecified and either the error terms have a more fat-tailed distribution in the diffusion model, or that the true model has discontinuities in it. That would mean there are jumps in the true price process. However, as we are employing the EMS method for simulating the terminal asset prices, we can not use this method, as our simulated price paths are not independent of each other. Furthermore, tracking all the residuals of all simulated paths would consume an excessive amount of computer memory and would significantly disturb the actual simulation process.

Nevertheless, a kind of a preliminary robustness check can be done by following Bates (2000), who compares the models by examining whether the implied parameters are consistent with those estimated from the time series data. In other words, he compares the implied volatility estimates with the sample standard deviations. One must keep in mind that the estimates implied by option prices correspond to the risk neutral distribution, whereas the time series estimates correspond to the true distribution. Therefore the implied parameters can not exactly match those estimated from time series data, even if estimation error is ignored. Having said that, according to Bates (1991) the differences between the parameters from the two distributions will not be significant, if and when the risk aversion coefficient of the representative agent is bounded within a reasonable range. In our data set the average implied volatilities for the Euro Stoxx 50 index and Nokia stock were 27.33% and 67.59%, respectively. The average sample standard deviations from the ten-year period March 29, 1993, to March 28, 2003, on the other hand, calculated from daily net returns using a 20-day sample and then annualized, averaged 19.43% and 48.22%. So it seems that our implied volatility estimates were somewhat higher, but nevertheless reasonable, given the data period of high volatility. Furthermore, the differences between the Nokia volatility and index volatility were pretty much of the same magnitude regardless of the calculation method.

4.3 Pricing Performance

While we have minimized the in-sample pricing error in the previous section, it is not sufficient to differentiate between models. A model may perform very well in terms of fitting the sample, but might still be useless when applied out of sample. This can happen, if the model does not capture the actual factors driving asset price, but only succeeds in matching the limited number of observations in the sample. An extreme, and very simple, example would be that if the sample consisted of only one option price, the best model to fit this sample would be just a scalar number equal to the price. The pricing error would be zero, but the 'model' would not tell us anything about the actual price process and would not be helpful in pricing other options. This problem tends to arise especially with the more complex models, as any sample can be approximated more closely by adding more independent variables. This issue is commonly known as *overfitting*. To control for this hazard, we will also calculate the pricing errors for a different time period than the one used for estimation. We calculate the pricing error as the sample averages of absolute and percentage errors.

4.4 Dynamic Hedging Performance

In this part we will be constructing a delta-neutral hedge and compare the models based on their respective hedging errors. The comparison of models is done in the spirit of Bakshi et al (1997). More specifically, their discussion on delta-neutral hedging (pp. 2036-2042) forms the basis for our analysis.[15]

To be able to analyze the hedging ability of an option-pricing model, we have to define the replicating portfolio, which consists of $X_S(t)$ units of the underlying stock to control for price risk, $X_C(t)$ units of another call option with a different strike price $\widetilde{K}$ to control for volatility risk and $X_0(t)$, the residual cash position. Then, as jumps in price and volatility processes are left uncontrolled for, as discussed earlier, $C(t,\tau,K)$, where t is time, τ time to maturity and K the strike price, is replicated by the portfolio $X_0(t)+$

[15] See also section 2.6 of this thesis for more discussion on the topic.

$X_S(t)+X_C(t)$. Bakshi et al (1997, p. 2038) have derived the following solution for these positions $X_i(t)$:

$$X_C(t)=\frac{\Delta_V(t,\tau,K)}{\Delta_V(t,\tau,\widetilde{K})} \tag{4.3}$$

$$X_S(t)=\Delta_S(t,\tau,K)-\Delta_S(t,\tau,\widetilde{K})X_C(t) \tag{4.4}$$

$$X_0(t)=C(t,\tau,K)-X_S(t)S(t)-X_C(t,\tau,\widetilde{K})C(t,\tau,\widetilde{K}) \tag{4.5}$$

where the primitive deltas, Δ_S and Δ_V, are as defined in equations (2.15) and (2.16) as well as in appendix A.

To examine the hedging effectiveness, at time t the call option is shorted and the hedging portfolio is established as described above. After the hedge interval Δt, which is the time after which the hedging portfolio is rebalanced, the hedging error $H(t+\Delta t)$ is computed by

$$\begin{aligned} H(t+\Delta t)=X_0e^{r\Delta t}+X_S(t)S(t+\Delta t) \\ +X_C(t)C(t+\Delta t,\tau-\Delta t,\widetilde{K})-C(t+\Delta t,\tau-\Delta t,K) \end{aligned} \tag{4.6}$$

Just like when studying the pricing performance, this procedure is carried out for all options in the data set and the average absolute and relative hedging errors are calculated. Note that for the BS model X_C is zero by definition, as the assumption is that there is no volatility risk to be hedged.

For a numerical example of constructing the delta-neutral hedging portfolio, see appendix B.

5 Results

In this chapter the results of the empirical simulation study will be presented. The discussion is divided into three parts: firstly, the parameter estimates are presented and it is discussed whether a given model can immediately be categorized as misspecified. Secondly, the comparison of models in terms of their pricing ability is conducted. Thirdly, the hedging ability-based comparison is left in the final sub-section.

5.1 Estimates of the Model Parameters

The parameter estimates for all the models under scrutiny are depicted in tables 2 and 3 for options on DJ Euro Stoxx 50 index and Nokia stock, respectively. It is to be noted that even though the estimation procedure was based on minimizing the sum of squared pricing errors, denoted SSE, the absolute SSE values are not very informative when comparing different tables against each other. This is caused by the fact that the SSEs were calculated from absolute errors, so the more expensive options are bound to have a bigger effect on the SSE than cheaper ones. This results in the SSEs of index options being systematically larger, as the options on the index are almost always more expensive than the options on Nokia stock.

Within a single table, however, the SSEs of different models are somewhat more informative as in principle the smaller the SSE, the better the fit within the sample. In general it seems to be the case that the more complicated the model, the smaller the SSE and therefore the better the fit. It is also noteworthy that the order of the models is different in the two tables, as the model with t-distributed price innovations seems to perform worse in the index options than in the Nokia options. On the other hand, with the index options including jumps in the volatility process seems to improve the fit much more than with the Nokia options.

Correlation coefficient between the innovations of asset price and volatility processes has, as discussed in section 3.1.1, been perhaps the most controversial parameter in early studies about stochastic volatility. Our estimates of the correlation coefficient range from -0.28 to -0.41 for the index and from -0.37 to -0.55 for the Nokia stock, depending on the model at hand. Of course, in the SVCJ model the correlation between diffusions is smaller, as also the jump processes are correlated and this contributes to the total correlation between the two processes. On average, our estimates of the index correlation coefficient are between the estimates of Eraker et al. (2002) for the S&P 500 index and Nasdaq 100 index. Their estimated correlations for S&P 500 were on average more negative than our estimates for DJ Euro Stoxx 50, whereas their estimates for the correlation coefficient of the Nasdaq 100 were on average slightly closer to zero. Also the results of Bakshi et al. (1997), even though in part their compared models differ from those studied here, suggested that the correlation coefficients for the S&P 500 index are more negative than what, according to our estimates, the coefficients for the DJ Euro Stoxx 50 index are. The estimate related to the basic SV model for Nokia stock seems to be considerably more negative than any of the other estimates. This can, in part, be also caused by estimation error as for Nokia stock the SV model seems not to provide as good a fit as for some other models.

Our estimates seem to be of a reasonable magnitude when compared with Eraker et al. (2002), as well as other studies. What is most different in the results of Eraker et al. (2002) as opposed to ours, is that they observed a correlation coefficient between jumps in the two processes as –0.6, whereas our estimate for the DJ Euro Stoxx index was only –0.25. However, as not only the underlying index but also the time period is different, estimates can not be directly compared with those from other studies, although some information can naturally be inferred.

The other parameter estimates are closer to the ones obtained by Eraker et al. (2002) and, in relevant parts, also by Bakshi et al. (1997, 2000), Bates (2000), Pan (2002), Andersen et al. (2002) and Liesenfeld and Jung (2001). Of course there are some differences, but on aggregate the estimates would seem to be roughly in line with earlier empirical studies. Therefore it can be argued that initially all the models would seem to imply more or less reasonable estimates for the parameters.

Table 2, Parameter Estimates for Options on DJ Euro Stoxx 50

In the table are depicted the parameter estimates for the index options for all the models studied. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility with normally distributed returns, Stochastic Volatility with t-distributed returns, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. The parameters of the volatility process, κ, θ, σ_v and ρ, are respectively the speed of adjustment, drift, volatility of volatility and the correlation coefficient between returns and volatility changes. λ_y and λ_v are frequencies of jumps per year in the logarithmic prices and in volatility, respectively. In the SVCJ model, however, λ_y and λ_v are restricted to being equal. μ_y is the expected jump size in log prices, σ_y the standard deviation of the jump in log prices, $1/\mu_v$ the expected jump size in volatility and ρ_J the correlation coefficient between jumps in log prices and volatility. In SVT, df is the degree of freedom of the t-distribution. SSE is the value of the optimization equation, i.e. the sum of squared pricing errors of the respective models as calculated from the data from July 2000.

DJ Euro Stoxx 50					
Parameters	SV	SVT	SVJ	SVIJ	SVCJ
κ	1.2025	1.2639	1.2003	1.3863	1.3487
θ	0.0453	0.0367	0.0345	0.0284	0.0247
σ_v	0.3019	0.3611	0.3799	0.3939	0.3746
ρ	-0.3979	-0.2807	-0.4003	-0.4139	-0.3479
λ_y			0.0061	0.0079	0.0015
λ_v				0.0049	0.0015
μ_y			-0.0502	-0.0511	-0.1087
μ_v				1.4278	2.4282
σ_y			0.0506	0.0520	0.0526
ρ_J					-0.2526
df		11.3976			
mean error	-0.2213	0.7428	0.3083	0.4539	1.6345
median error	2.2055	4.6406	3.2462	2.4598	0.3983
SSE	53342.17	57138.06	44781.13	42967.33	35419.68

Table 3, Parameter Estimates for Options on Nokia Stock

In the table are depicted the parameter estimates for Nokia options for all the models studied. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility with normally distributed returns, Stochastic Volatility with t-distributed returns, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. The parameters of the volatility process, κ, θ, σ_v and ρ, are respectively the speed of adjustment, drift, volatility of volatility and the correlation coefficient between returns and volatility changes. λ_y and λ_v are frequencies of jumps per year in the logarithmic prices and in volatility, respectively. In the SVCJ model, however, λ_y and λ_v are restricted to being equal. μ_y is the expected jump size in log prices, σ_y the standard deviation of the jump in log prices, $1/\mu_v$ the expected jump size in volatility and ρ_J the correlation coefficient between jumps in log prices and volatility. In SVT, df is the degree of freedom of the t-distribution. SSE is the value of the optimization equation, i.e. the sum of squared pricing errors of the respective models as calculated from the data from July 2000.

Nokia					
Parameters	SV	SVT	SVJ	SVIJ	SVCJ
κ	1.3568	1.641	1.4805	1.3587	1.2834
θ	0.3190	0.241	0.2856	0.2649	0.2576
σ_v	0.6457	0.745	0.6728	0.5875	0.5817
ρ	-0.5572	-0.379	-0.3966	-0.3660	-0.3165
λ_y			0.0083	0.0070	0.0017
λ_v				0.0058	0.0017
μ_y			-0.0592	-0.0590	-0.0829
μ_v				1.4170	1.9333
σ_y			0.0812	0.1126	0.0952
ρ_J					-0.3401
df		8.413			
mean error	-0.0044	-0.0186	-0.0568	-0.0965	-0.1758
median error	-0.0675	-0.0717	-0.1016	-0.1585	-0.2241
SSE	24.1241	17.8450	17.2234	18.0337	21.8587

For the SVT model, Liesenfeld and Jung (2000) argue that to benefit from the usage of the t-distribution, the degree of freedom (df) must be larger than four, otherwise the kurtosis[16] of the distribution does not exist (pp. 142-143). Note that if df $\rightarrow \infty$, the t-distribution approaches the normal distribution. Liesenfeld and Jung (2000) also studied what the optimal degrees of freedom for different assets are. For the S&P500 – index

[16] For t-distributed error u, kurtosis is defined as $E(u^4) = 3(df-2)/(df-4)$. If df ≤ 4, the kurtosis does not exist. As the distribution approaches the normal distribution, i.e. df $\rightarrow \infty$, the kurtosis approaches 3.

their estimate was 10.7 and for three individual stocks (IBM listed in NYSE, Siemens and Daimler-Benz listed at Deutsche Börse) the estimates ranged from 8.24 to 8.73, averaging 8.6. Our estimated degrees of freedom for DJ Euro Stoxx 50 and Nokia, 11.4 and 8.4 respectively, are therefore in line with their results and yield a kurtosis of 3.81 for the index and 4.36 for the Nokia stock. When comparing these implied kurtosis figures with the historical ones calculated from the data period March 29 1993 to March 28 2003 for the daily net returns of DJ Euro Stoxx 50 and Nokia stock (see section 4.1), it can be seen that they are roughly of the same magnitude. The historical kurtosis has been 3.44 for the index returns and 5.30 for the Nokia returns.

5.2 Pricing Performance

In this section the pricing results for the period not used for the parameter estimation, i.e. August 2000 to June 2002, are reported. This kind of comparison is essential to perform if one wants to get a more realistic picture of the models' pricing abilities, the in-sample comparison alone can lead significantly astray. In tables 4 and 5 both absolute and relative pricing errors of the different models are depicted for the options on the DJ Euro Stoxx 50 index and on Nokia stock, respectively. The tables show the averages of the errors from the out-of-sample data period August 2000 to June 2002. The graphical representations of the distributions of the absolute error terms for each individual model are given in appendix C.

At first glance the errors seem to be rather large, especially on the uppermost rows. This is to be expected, as these rows show the errors for deep OTM options, which have traditionally been the most difficult to match (see e.g. Bakshi et al., 1997). However, as a whole, our pricing errors are in general further away from zero than for example those of Bakshi et al. (1997). There are several factors causing our models to perform worse than in comparable studies, the most important of which is probably our rather crude method for estimating the volatility.

Table 4, Out-of-Sample Pricing Errors, DJ Euro Stoxx 50 Index

In the table are depicted the out-of-sample pricing errors for all the models studied. The underlying asset is the DJ Euro Stoxx 50 index. Out-of-sample data spans the time period from August 2000 to June 2002. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility, Stochastic Volatility with t-distributed innovations, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. n is the number of observations in each category. Both the absolute pricing errors and the relative, or percentage, pricing errors are presented. The reported absolute pricing error is the sample average of the absolute pricing differences between the market price and the model price, in each moneyness – maturity category. The percentage pricing error is calculated as the absolute error dividend by the market price.

DJ Euro Stoxx 50

		Absolute Pricing Errors				Relative Pricing Errors			
Moneyness		Days to Expiry				Days to Expiry			
S/K	Model	<60	60-180	>180	TOTAL	<60	60-180	>180	TOTAL
<0.94	n	931	1214	1112	3257				
	BS	8.4609	44.2822	74.7325	44.4391	87.62 %	161.98 %	197.78 %	152.95 %
	SV	-4.8911	-2.9640	11.0974	1.2860	-2.96 %	11.53 %	39.68 %	17.00 %
	SVT	-2.7823	-1.1358	11.3662	2.6620	21.61 %	16.71 %	39.99 %	26.06 %
	SVJ	-4.8262	-3.4081	9.5074	0.5961	-3.03 %	8.71 %	33.10 %	13.68 %
	SVIJ	-5.2108	-4.5358	9.2206	-0.0321	-6.45 %	4.65 %	32.55 %	11.00 %
	SVCJ	-7.2618	-9.0522	5.9830	-3.4072	-25.91 %	-11.64 %	16.05 %	-6.27 %
0.94-0.97	n	719	327	194	1240				
	BS	8.4771	36.8852	66.7198	25.0808	15.02 %	27.29 %	27.52 %	20.21 %
	SV	-3.9709	-11.3273	-3.1806	-5.7872	-9.25 %	-5.23 %	-1.02 %	-6.90 %
	SVT	-0.2991	-9.0294	-3.9533	-3.1730	5.32 %	-3.24 %	-1.26 %	2.04 %
	SVJ	-3.0408	-10.0112	-2.5726	-4.8057	-5.81 %	-4.33 %	-0.78 %	-4.64 %
	SVIJ	-3.3787	-10.7312	-2.4600	-5.1739	-6.15 %	-4.81 %	-0.71 %	-4.94 %
	SVCJ	-6.1511	-12.9892	5.2156	-6.1760	-15.90 %	-7.18 %	2.02 %	-10.80 %
0.97-1.00	n	790	381	268	1439				
	BS	3.0151	32.6217	56.3128	20.7801	-1.84 %	17.13 %	18.35 %	6.94 %
	SV	-9.4947	-17.0348	-7.2451	-11.0721	-15.24 %	-7.11 %	-2.08 %	-10.63 %
	SVT	-5.4152	-14.4899	-6.0675	-7.9394	-7.47 %	-5.66 %	-1.71 %	-5.92 %
	SVJ	-7.6024	-14.7531	-4.5577	-8.9286	-12.33 %	-5.89 %	-1.23 %	-8.55 %
	SVIJ	-7.7222	-15.0176	-3.7781	-8.9192	-12.27 %	-6.00 %	-1.00 %	-8.51 %
	SVCJ	-10.1047	-14.2654	7.4432	-7.9382	-16.28 %	-5.96 %	2.44 %	-10.06 %
1.00-1.03	n	383	189	152	724				
	BS	-5.1732	29.2443	60.7009	17.6414	-7.03 %	11.65 %	16.63 %	2.81 %
	SV	-18.8454	-24.3583	-13.8604	-19.2380	-15.78 %	-8.72 %	-3.66 %	-11.39 %
	SVT	-14.8418	-21.7377	-13.0910	-16.2744	-12.28 %	-7.55 %	-3.45 %	-9.19 %
	SVJ	-16.0672	-21.2151	-11.2308	-16.3957	-13.49 %	-7.39 %	-2.93 %	-9.68 %
	SVIJ	-15.8516	-20.1159	-10.3059	-15.8005	-13.18 %	-6.93 %	-2.70 %	-9.35 %
	SVCJ	-16.8448	-16.6746	4.5898	-12.3003	-14.27 %	-5.59 %	1.33 %	-8.73 %
1.03-1.06	n	72	69	36	177				
	BS	-11.7482	19.9485	43.4358	11.8320	-6.53 %	6.16 %	10.28 %	1.84 %
	SV	-24.9713	-36.8270	-19.9428	-28.5702	-12.37 %	-11.06 %	-4.63 %	-10.29 %
	SVT	-21.1965	-34.5753	-17.6052	-25.6815	-10.55 %	-10.37 %	-3.99 %	-9.14 %
	SVJ	-20.9856	-32.9342	-15.9659	-24.6226	-10.43 %	-9.85 %	-3.69 %	-8.83 %
	SVIJ	-20.3190	-32.0817	-14.2635	-23.6728	-10.11 %	-9.61 %	-3.27 %	-8.52 %
	SVCJ	-19.1290	-24.1830	2.6663	-16.6662	-9.65 %	-7.20 %	0.76 %	-6.58 %
>1.06	n	52	58	42	152				
	BS	-17.3492	2.2799	31.0880	3.5248	-5.23 %	1.10 %	5.89 %	0.26 %
	SV	-29.5070	-34.6012	-20.8212	-29.0508	-8.89 %	-7.41 %	-3.58 %	-6.86 %
	SVT	-27.0709	-33.2511	-21.1756	-27.8002	-8.15 %	-7.11 %	-3.61 %	-6.50 %
	SVJ	-25.1474	-30.8349	-16.8469	-25.0241	-7.63 %	-6.64 %	-2.89 %	-5.94 %
	SVIJ	-24.6819	-29.3038	-15.4592	-23.8971	-7.50 %	-6.29 %	-2.67 %	-5.70 %
	SVCJ	-17.9695	-15.0318	6.2682	-10.1513	-5.68 %	-3.33 %	0.99 %	-2.94 %
All	n	2947	2238	1804	6989				
Moneyness-	BS	4.2839	38.1076	68.3115	31.6416	29.68 %	95.97 %	129.34 %	76.63 %
Categories	SV	-8.6391	-10.2521	3.3716	-6.0554	-9.79 %	3.01 %	23.56 %	2.92 %
	SVT	-5.3280	-8.1657	3.7324	-3.8980	4.13 %	6.48 %	23.81 %	9.96 %
	SVJ	-7.3491	-9.4292	3.2496	-5.2794	-7.82 %	1.99 %	19.75 %	2.44 %
	SVIJ	-7.5326	-10.0323	3.3449	-5.5254	-8.92 %	-0.24 %	19.48 %	1.19 %
	SVCJ	-9.4772	-11.7801	5.9404	-6.2351	-18.62 %	-9.16 %	10.62 %	-8.04 %

Table 5, Out-of-Sample Pricing Errors, Nokia Stock

In the table are depicted the out-of-sample pricing errors for all the models studied. The underlying asset is the Nokia stock. Out-of-sample data spans the time period from August 2000 to June 2002. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility, Stochastic Volatility with t-distributed innovations, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. n is the number of observations in each category. Both the absolute pricing errors and the relative, or percentage, pricing errors are presented. The reported absolute pricing error is the sample average of the absolute pricing differences between the market price and the model price, in each moneyness – maturity category. The percentage pricing error is calculated as the absolute error dividend by the market price.

Nokia									
		Absolute Pricing Errors				Relative Pricing Errors			
Moneyness		Days to Expiry				Days to Expiry			
S/K	Model	<60	60-180	>180	TOTAL	<60	60-180	>180	TOTAL
<0.94	n	1236	1045	638	2919				
	BS	0.0391	0.3902	0.9969	0.3742	1.00 %	42.38 %	70.90 %	31.09 %
	SV	-0.1971	-0.1558	0.1901	-0.0977	-34.07 %	-7.31 %	16.24 %	-13.49 %
	SVT	-0.1956	-0.1635	0.1728	-0.1036	-32.93 %	-7.87 %	14.68 %	-13.55 %
	SVJ	-0.2133	-0.1823	0.1477	-0.1233	-37.23 %	-10.26 %	12.97 %	-16.60 %
	SVIJ	-0.2267	-0.2233	0.0939	-0.1554	-38.66 %	-13.15 %	9.15 %	-19.08 %
	SVCJ	-0.2558	-0.2722	0.0579	-0.1931	-43.99 %	-17.02 %	4.47 %	-21.98 %
0.94-0.97	n	248	135	45	428				
	BS	0.0152	0.3289	0.9859	0.2162	-5.20 %	10.76 %	18.85 %	2.37 %
	SV	-0.2610	-0.2360	0.1729	-0.2071	-22.55 %	-6.95 %	3.00 %	-14.94 %
	SVT	-0.2579	-0.2377	0.1184	-0.2120	-21.60 %	-6.96 %	2.08 %	-14.49 %
	SVJ	-0.2748	-0.2627	0.1426	-0.2271	-23.99 %	-7.71 %	2.46 %	-16.08 %
	SVIJ	-0.2967	-0.3045	0.0410	-0.2636	-25.22 %	-9.09 %	0.69 %	-17.41 %
	SVCJ	-0.3316	-0.3437	0.0718	-0.2930	-27.87 %	-10.22 %	0.90 %	-19.28 %
0.97-1.00	n	220	124	55	399				
	BS	-0.0118	0.3656	1.1157	0.2609	-6.01 %	10.46 %	20.77 %	2.80 %
	SV	-0.3206	-0.2477	0.0944	-0.2407	-20.04 %	-7.03 %	0.90 %	-13.11 %
	SVT	-0.3241	-0.2532	0.0633	-0.2487	-20.08 %	-7.23 %	0.24 %	-13.29 %
	SVJ	-0.3443	-0.2832	0.0354	-0.2730	-21.40 %	-7.98 %	-0.18 %	-14.30 %
	SVIJ	-0.3639	-0.3092	-0.0575	-0.3046	-22.31 %	-8.67 %	-2.00 %	-15.27 %
	SVCJ	-0.4018	-0.3409	-0.0369	-0.3325	-24.51 %	-9.65 %	-1.54 %	-16.72 %
1.00-1.03	n	142	64	29	235				
	BS	-0.0927	0.3614	1.0302	0.1696	-6.15 %	8.54 %	16.55 %	0.65 %
	SV	-0.3564	-0.2930	0.0485	-0.2891	-16.95 %	-6.47 %	0.74 %	-11.92 %
	SVT	-0.3568	-0.3154	0.0301	-0.2978	-16.82 %	-6.96 %	0.30 %	-12.02 %
	SVJ	-0.3727	-0.3208	0.0408	-0.3075	-17.63 %	-7.05 %	0.77 %	-12.48 %
	SVIJ	-0.3847	-0.3648	-0.0579	-0.3389	-18.26 %	-8.11 %	-1.09 %	-13.38 %
	SVCJ	-0.4089	-0.3873	-0.0122	-0.3541	-19.30 %	-8.62 %	-0.21 %	-14.03 %
1.03-1.06	n	96	38	18	152				
	BS	-0.1346	0.2986	0.9004	0.0962	-7.19 %	6.87 %	14.84 %	-1.07 %
	SV	-0.3739	-0.3517	0.0044	-0.3233	-15.86 %	-8.03 %	-1.15 %	-12.16 %
	SVT	-0.3676	-0.3731	0.0136	-0.3239	-15.48 %	-8.46 %	-1.13 %	-12.02 %
	SVJ	-0.3753	-0.3851	-0.0077	-0.3342	-15.88 %	-8.61 %	-1.57 %	-12.37 %
	SVIJ	-0.3950	-0.4317	-0.0830	-0.3672	-16.48 %	-9.64 %	-2.22 %	-13.08 %
	SVCJ	-0.4092	-0.4442	-0.1056	-0.3820	-17.06 %	-9.91 %	-2.90 %	-13.60 %
>1.06	n	216	96	47	359				
	BS	-0.1560	0.0747	0.7959	0.0303	-3.95 %	1.80 %	11.36 %	-0.41 %
	SV	-0.3221	-0.3931	-0.0822	-0.3097	-7.94 %	-6.70 %	-1.44 %	-6.76 %
	SVT	-0.3230	-0.3966	-0.1260	-0.3169	-7.94 %	-6.75 %	-2.04 %	-6.85 %
	SVJ	-0.3209	-0.3969	-0.1270	-0.3158	-7.91 %	-6.77 %	-1.99 %	-6.83 %
	SVIJ	-0.3294	-0.4258	-0.1718	-0.3346	-8.11 %	-7.32 %	-2.68 %	-7.19 %
	SVCJ	-0.3240	-0.4043	-0.1475	-0.3223	-8.06 %	-6.98 %	-2.29 %	-7.02 %
All	n	2158	1502	832	4492				
Moneyness-	BS	-0.0048	0.3590	0.9919	0.3015	-1.76 %	31.97 %	58.30 %	20.64 %
Categories	SV	-0.2477	-0.1966	0.1585	-0.1554	-26.77 %	-7.20 %	12.60 %	-12.93 %
	SVT	-0.2469	-0.2043	0.1374	-0.1615	-25.98 %	-7.64 %	11.26 %	-12.95 %
	SVJ	-0.2622	-0.2226	0.1174	-0.1786	-28.92 %	-9.44 %	9.95 %	-15.21 %
	SVIJ	-0.2769	-0.2619	0.0569	-0.2101	-30.06 %	-11.74 %	6.69 %	-17.13 %
	SVCJ	-0.3031	-0.3020	0.0348	-0.2401	-33.73 %	-14.62 %	4.22 %	-20.31 %

Another factor could be, although to a lesser extent, simulation error. Bakshi et al (1997) use the same estimation method but they also study in part different models than we do and are therefore able to apply closed-form pricing formulas for all models studied. Therefore there is, at least theoretically, no random error in their price estimates, whereas our estimates are bound to have some random error in them. This fact also explain why the (closed-form) BS model performs relatively better in our comparison with the other models than in other studies.

Bakshi et al (1997) document systematic overpricing of OTM and underpricing of ITM options for all the models they studied, and our results seem to support this finding, at least as far as the index options are considered. It can be seen that for Nokia options, all but the long-term OTM options are underpriced. This could imply that the parameter estimates are not optimal for the out-of-sample data but produce too low prices for the options. It is interesting, however, that the long-term OTM options are priced above their true market prices thus making an exception to the pattern.

Also in general, for options both on the index and the stock, options with maturities more than 180 days the pricing errors are significantly lower than for short-term options. This holds true for all the simulated models, whereas for the BS the opposite is true. This tendency seems to give support to the claim that when the maturity of an option is extended, other factors than just the price risk increase in importance, as there is more time for volatility to diffuse and jumps to take place. Furthermore, when comparing the models allowing jumps with the SV and SVT models, the jumps seem to become more important with maturity. This tendency, although very small, implies contradiction to the results of Das and Sundaram (1999), who argue that in the long run jumps become less important in relation with the stochastic volatility. However, they only compared pure jump-diffusion models with pure SV-models and therefore their results are not directly comparable to ours. Furthermore, there is a pattern in the results of Bakshi et al. (1997) that is not too different from our results.

The main focus of Bakshi et al. (2000) was to examine the performance of the models when the maturities are extended, although in their paper long-term options had maturities more than a year, whereas we have focused only on options with less than a year running time left. Nevertheless, Bakshi et al. (2000) showed that theoretically options with longer maturities should be better able to differentiate between models and their empirical results implied that the rankings of models can indeed become different when long-term options are studied. This is also what we witnessed with our data and our models, as especially the Black-Scholes models performed systematically worse on options with longer maturities.

What comes to the pricing performance of individual models, it can be said that even if the BS model is the only closed-form model in comparison, it still performs the worst on aggregate. However, when pricing ITM options with shorter maturities the BS still outperforms the other models. The tendency of the BS to perform relatively better with short-term ITM options was observed also by Bakshi et al. (1997, 2000), even though in their comparisons it was the worst model also in that category.

It is surprising and somewhat disappointing that the SVT model does not seem to perform any better than the regular SV model. The performance in pricing the Nokia options is almost identical, but with index options there are differences, but depending on the category both models occasionally get the upper hand. It is interesting that the order of the two models is different when comparing the absolute pricing errors versus the relative ones. This can be explained by the different weights of cheap (short-term, OTM) and more expensive (long-term, ITM) options in computing the average errors.

Prior studies (e.g. Bakshi et al. 1997, 2000, Bates 1996a, 2000, Pan 2002, Andersen et al. 2002) have documented strong evidence in favor of including jumps in the price process. Our results are not as clear, as evidence on the index options seems to slightly support the importance of jumps, especially when pricing OTM options, but in the stock option case this does not hold true. It can be suspected that our jump parameter estimates for the Nokia price and/or volatility process are not accurate enough and that this would cause the contradicting evidence. However, the result can not be dismissed right away and the difference between stock vs. index options calls for further research. Especially, as initially our intuitive assumption was that the jumps would play a bigger part in pricing the more volatile stock options. Perhaps the bigger instantaneous volatility holds the key for this question, as it can be argued that if the volatility is in general on a higher level, as is the case with the Nokia stock compared with the index, the relative importance of jumps in the price process decreases. Possibly the same holds true also for the volatility process, i.e. the volatility of volatility and jumps in the volatility process.

Eraker et al. (2002) report strong evidence for the importance of jumps in the volatility process. Our results give some support to this claim, but the differences against the SVJ model are quite small. When comparing the SVIJ and SVCJ models against each other, we must agree with Eraker et al. (2002) who found little support for the correlation between jump sizes to returns and jump sizes to volatility. However, when pricing OTM options it does seem to perform a little better than the SVIJ model. For the index options, this is true already for options close to ATM, but for the stock options only deep OTM options are better priced by the SVCJ model.

5.3 Dynamic Hedging Performance

In this section the hedging performance of the examined models is reported similarly to the pricing performance in the previous section. The hedging performance is measured through the errors that result when hedging an option using a hedging strategy given by the pricing model. The detailed hedging portfolio was described in section 4.4, and appendix B presents a numerical example of hedging and of computing the hedging errors. In this study a hedging period of one day was used. This means that the hedging error, i.e. the difference between the market values of the hedging portfolio and the hedged option, was calculated one trading day after implementing the hedge. This period of one day is fairly common in the literature (for instance Bakshi et al., 1997, Nandi, 1998) although also other periods have sometimes been used (for example Bakshi et al., 2000).

In tables 6 and 7 the hedging errors for the options on the Euro Stoxx 50 index and the Nokia stock, respectively, are presented. The graphical representations of the absolute hedging errors can be found in appendix D.

Table 6, Out-of-Sample Hedging Errors, DJ Euro Stoxx 50 Index

In the table are depicted the out-of-sample hedging errors for all the models studied. The underlying asset of the hedged option is the DJ Euro Stoxx 50 index. Out-of-sample data spans the time period from August 2000 to June 2002. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility, Stochastic Volatility with t-distributed innovations, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. n is the number of observations in each category. In Panel A, the absolute monetary hedging errors, calculated as the difference between the market price of the hedged option and the hedging portfolio one day after the hedge is constructed, are presented, whereas in Panel B the relative, or percentage, hedging errors. The reported absolute hedging error is the sample average of the monetary errors, in each moneyness – maturity category. The percentage pricing error is the sample average of the absolute errors divided by the corresponding market prices.

DJ Euro Stoxx 50

		Absolute Pricing Errors				Relative Pricing Errors			
Moneyness		Days to Expiry				Days to Expiry			
S/K	Model	<60	60-180	>180	TOTAL	<60	60-180	>180	TOTAL
<0.94	n	655	751	599	2005				
	BS	0.0208	0.0790	0.1161	0.0711	68.86 %	-9.77 %	-93.38 %	-9.06 %
	SV	-1.9247	-2.3489	0.0076	-1.5063	-40.90 %	-5.40 %	-3.37 %	-16.39 %
	SVT	-2.1342	-4.5147	1.0252	-2.0820	-460.74 %	-3.08 %	-0.92 %	-151.94 %
	SVJ	7.6025	-1.1716	-5.5811	0.3774	-22.43 %	-11.81 %	-7.18 %	-13.90 %
	SVIJ	0.4186	2.3679	-4.3310	-0.2702	24.52 %	0.02 %	-5.12 %	6.49 %
	SVCJ	-30.5616	306.1106	59.1632	122.3491	-294.02 %	322.44 %	73.42 %	46.66 %
0.94-0.97	n	442	189	98	729				
	BS	-1.9589	0.6646	1.0975	-0.8679	-11.97 %	0.49 %	0.33 %	-7.08 %
	SV	-4.4909	3.5175	-1.8672	-2.0620	-16.09 %	2.57 %	-0.82 %	-9.20 %
	SVT	-1.4776	-8.1636	-3.5067	-3.4838	-17.76 %	-5.53 %	-1.59 %	-12.41 %
	SVJ	-1.3052	9.3316	-0.4902	1.5621	-11.25 %	4.44 %	-0.15 %	-5.69 %
	SVIJ	2.3935	11.0244	-0.7695	4.2059	20.99 %	9.72 %	-0.23 %	15.21 %
	SVCJ	-23.5898	168.8592	-794.0849	-77.2738	137.00 %	213.85 %	-389.68 %	86.12 %
0.97-1.00	n	457	207	119	783				
	BS	-0.0816	0.7144	2.8924	0.5808	-0.71 %	0.44 %	1.05 %	-0.14 %
	SV	-1.5505	-0.4161	0.3337	-0.9642	-12.88 %	-0.47 %	-0.02 %	-7.64 %
	SVT	0.8785	1.9823	4.7571	1.7597	-2.14 %	1.36 %	1.59 %	-0.65 %
	SVJ	-9.0328	2.3012	4.8886	-3.9207	-43.37 %	0.75 %	1.87 %	-24.83 %
	SVIJ	0.1870	0.4056	-0.2366	0.1804	-0.32 %	0.66 %	-0.18 %	-0.04 %
	SVCJ	14.9467	-465.8413	-22.6748	-117.8759	50.11 %	-262.00 %	-8.30 %	-41.28 %
1.00-1.03	n	258	117	65	440				
	BS	1.0876	-0.2316	-3.2994	0.0887	2.28 %	0.02 %	-0.95 %	1.20 %
	SV	2.6787	2.2258	-14.7716	-0.0196	1.66 %	0.59 %	-4.18 %	0.51 %
	SVT	0.7812	-3.7578	-19.9857	-3.4936	2.76 %	-1.76 %	-5.79 %	0.29 %
	SVJ	7.0663	3.7091	-5.6004	4.3024	3.58 %	1.59 %	-1.09 %	2.36 %
	SVIJ	2.8605	-7.4079	-7.1857	-1.3541	1.07 %	-2.96 %	-2.01 %	-0.46 %
	SVCJ	49.5894	-47.3195	-24.2042	12.9191	56.12 %	-20.36 %	-3.49 %	26.98 %
1.03-1.06	n	61	57	22	140				
	BS	2.6578	2.5018	8.1968	3.4647	2.11 %	0.72 %	2.06 %	1.54 %
	SV	1.6310	-2.3902	-2.4422	-0.6463	1.17 %	-0.77 %	-0.54 %	0.11 %
	SVT	2.0735	0.6213	1.7655	1.4339	0.47 %	0.09 %	0.58 %	0.33 %
	SVJ	0.7792	-11.3237	-3.7579	-4.8614	0.41 %	-3.88 %	-0.85 %	-1.54 %
	SVIJ	-2.2224	-5.0876	6.1796	-2.0686	-1.47 %	-1.98 %	1.77 %	-1.17 %
	SVCJ	60.8566	1610.4722	-29.5259	677.5686	35.38 %	533.77 %	-5.66 %	231.85 %
>1.06	n	44	48	30	122				
	BS	-4.1596	7.1175	-6.4868	-0.2949	-1.14 %	1.32 %	-0.95 %	-0.12 %
	SV	5.2296	1.4189	-0.4676	2.3294	1.58 %	0.27 %	-0.13 %	0.64 %
	SVT	-0.6143	1.5157	-2.7940	-0.3122	-0.40 %	0.26 %	-0.34 %	-0.13 %
	SVJ	1.7108	-0.2510	0.2852	0.5884	0.55 %	-0.02 %	-0.22 %	0.14 %
	SVIJ	0.3250	26.6301	66.5819	26.9672	-0.03 %	5.85 %	9.90 %	4.73 %
	SVCJ	14.6933	16.4953	207.6816	62.8585	4.46 %	10.36 %	37.44 %	14.89 %
All	n	1917	1369	933	4219				
Moneyness-	BS	-0.3285	0.5770	0.3136	0.1073	20.95 %	-5.14 %	-59.83 %	-3.25 %
Categories	SV	-1.5303	-0.7254	-1.2504	-1.2072	-20.46 %	-2.65 %	-2.56 %	-6.47 %
	SVT	-0.7035	-3.5461	-0.5439	-1.5906	-161.65 %	-2.38 %	-0.96 %	-44.93 %
	SVJ	1.1584	0.8303	-3.4808	0.0260	-20.09 %	-5.78 %	-4.49 %	-7.24 %
	SVIJ	1.0612	2.9711	-1.1056	1.2018	13.24 %	1.32 %	-3.11 %	3.47 %
	SVCJ	-9.4772	-11.7801	-44.0217	48.5644	-48.15 %	187.64 %	5.98 %	24.35 %

Table 7, Out-of-Sample Hedging Errors, Nokia Stock

In the table are depicted the out-of-sample hedging errors for all the models studied. The underlying asset of the hedged option is the DJ Euro Stoxx 50 index. Out-of-sample data spans the time period from August 2000 to June 2002. SV, SVT, SVJ, SVIJ and SVCJ stand for Stochastic Volatility, Stochastic Volatility with t-distributed innovations, Stochastic Volatility with Jumps in returns, Stochastic Volatility with Independent jumps in returns and volatility, and Stochastic Volatility with Correlated Jumps in returns and volatility, respectively. n is the number of observations in each category. In Panel A, the absolute monetary hedging errors, calculated as the difference between the market price of the hedged option and the hedging portfolio one day after the hedge is constructed, are presented, whereas in Panel B the relative, or percentage, hedging errors. The reported absolute hedging error is the sample average of the monetary errors, in each moneyness – maturity category. The percentage pricing error is the sample average of the absolute errors divided by the corresponding market prices.

Nokia									
		Absolute Pricing Errors				Relative Pricing Errors			
Moneyness		Days to Expiry				Days to Expiry			
S/K	Model	<60	60-180	>180	TOTAL	<60	60-180	>180	TOTAL
<0.94	n	1037	809	414	2260				
	BS	-0.1140	-0.1089	-0.1387	-0.1189	-50.41 %	-24.21 %	4.99 %	-22.31 %
	SV	-0.5041	-0.2324	0.7070	-0.0609	-50.99 %	-40.03 %	-173.44 %	-80.83 %
	SVT	-0.1620	-7.8867	3.8863	-2.4488	-237.33 %	-496.51 %	479.06 %	-261.70 %
	SVJ	0.0749	0.0873	-0.0690	0.0807	3.64 %	-30.89 %	-35.41 %	-15.83 %
	SVIJ	0.2031	0.5048	0.3505	0.3497	-60.37 %	43.03 %	138.65 %	55.06 %
	SVCJ	-0.6263	0.7850	0.5752	0.0943	-23.53 %	-371.38 %	-140.20 %	-191.30 %
0.94-0.97	n	173	83	18	274				
	BS	-0.1940	0.0146	-0.2043	-0.1547	-20.98 %	-3.24 %	-4.34 %	-16.65 %
	SV	-0.2098	1.6834	-0.2197	0.4889	2.26 %	48.79 %	-8.14 %	9.07 %
	SVT	-5.6690	-1.1352	13.0114	1.2418	-856.68 %	-64.86 %	315.87 %	85.90 %
	SVJ	-0.7425	1.5659	0.0037	0.0511	-92.31 %	67.81 %	1.73 %	-45.67 %
	SVIJ	-1.6460	1.7457	-0.1152	0.5635	-90.45 %	94.80 %	-5.97 %	26.21 %
	SVCJ	-4.7595	-0.8728	3.1128	-4.0781	-427.17 %	-65.51 %	48.57 %	-379.13 %
0.97-1.00	n	146	59	21	226				
	BS	-0.1811	0.0300	-0.0693	-0.1305	-13.38 %	0.42 %	-1.14 %	-9.53 %
	SV	-0.1338	-2.6887	-0.0547	-0.9555	-332.21 %	-109.55 %	-5.82 %	-314.42 %
	SVT	-0.8469	1.8378	-2.0410	-0.6875	-4.93 %	65.42 %	-51.95 %	-46.88 %
	SVJ	1.5159	-6.8207	-0.3548	-1.3351	140.76 %	-192.33 %	-6.77 %	1.45 %
	SVIJ	0.3425	0.4766	0.4282	0.3769	39.20 %	22.17 %	8.85 %	20.54 %
	SVCJ	4.9090	1.7478	-32.5578	0.5092	320.03 %	89.59 %	-772.04 %	147.55 %
1.00-1.03	n	92	31	14	137				
	BS	-0.1350	0.1621	-0.2105	-0.1161	-11.15 %	2.87 %	-3.73 %	-9.10 %
	SV	-0.1115	0.1242	-7.1263	-0.9664	17.81 %	-2.72 %	-79.11 %	-7.43 %
	SVT	13.4196	-3.0868	1.5539	-0.6686	803.41 %	-97.71 %	13.84 %	-17.43 %
	SVJ	1.0278	3.1111	0.4315	0.6386	57.41 %	95.59 %	5.51 %	0.75 %
	SVIJ	1.3109	1.0116	-3.1608	0.3512	106.95 %	26.75 %	-34.16 %	22.31 %
	SVCJ	2.6305	-1.2994	-12.8935	-0.9518	127.38 %	-64.93 %	-119.29 %	10.55 %
1.03-1.06	n	61	20	5	86				
	BS	-0.1498	0.0848	-0.0228	-0.1294	-7.96 %	0.91 %	-1.13 %	-8.22 %
	SV	-0.0796	1.0603	1.6017	0.3927	59.98 %	32.86 %	28.46 %	56.97 %
	SVT	1.1425	-75.9251	216.4522	-6.2586	41.63 %	-1843.42 %	4099.66 %	-228.87 %
	SVJ	0.1548	-1.3745	4.9062	-0.2395	9.24 %	-65.50 %	97.99 %	-21.54 %
	SVIJ	-1.0521	3.1797	-21.2214	0.3462	-35.78 %	53.62 %	-306.47 %	30.59 %
	SVCJ	0.4472	-2.0432	-14.1003	-1.2391	-1.12 %	-78.80 %	-255.56 %	-33.57 %
>1.06	n	142	62	15	219				
	BS	-0.0260	0.0772	-0.0378	0.0006	-2.55 %	0.17 %	-1.19 %	-2.20 %
	SV	-0.0255	-1.6571	0.9914	-0.2122	212.96 %	-16.97 %	17.76 %	169.83 %
	SVT	0.4855	0.4752	-2.4931	0.3274	12.99 %	5.32 %	-52.23 %	7.20 %
	SVJ	0.0489	4.0140	2.6865	1.6721	-0.32 %	69.41 %	51.26 %	28.28 %
	SVIJ	-0.3323	0.8956	1.7723	0.1044	-4.17 %	15.37 %	27.22 %	2.59 %
	SVCJ	-5.7014	1.8331	2.9582	-3.4205	-152.06 %	22.25 %	51.22 %	-107.15 %
All	n	1651	1064	487	3202				
Moneyness-	BS	-0.1231	-0.0692	-0.1359	-0.1072	-36.16 %	-18.53 %	3.88 %	-11.09 %
Categories	SV	-0.2940	-0.2675	0.4327	-0.1746	-39.24 %	-33.16 %	-149.43 %	-24.73 %
	SVT	0.0622	-7.4726	5.8868	-1.5567	-191.84 %	-416.14 %	457.56 %	-76.79 %
	SVJ	0.1688	0.1090	0.0717	0.1342	8.40 %	-23.26 %	-27.59 %	-3.48 %
	SVIJ	-0.0094	0.6879	0.0580	0.2327	-39.70 %	44.02 %	114.74 %	5.32 %
	SVCJ	-0.7921	-11.7801	-1.2242	-0.3763	-37.65 %	-284.60 %	-155.16 %	-63.03 %

The first thing that one notices when looking at the tables presenting the hedging errors is that based on the aggregate results, whether aggregated over different times to maturity or over the moneyness categories or both, the Black-Scholes model seems to be performing much better than expected based on the results of e.g. Bakshi et al. (1997). This holds true for both tables and therefore for both underlying assets. However, when looking more closely at the mean errors in individual moneyness-maturity categories, it can be seen that often the BS model is not the best model, but one or more of the other models outperform it. But how is it that overall the BS still seems to be showing the best performance? Probably the biggest reason for this is the simulation error, which can have a bigger effect on hedging errors than comparable pricing errors, as also the partial derivatives of the option prices and therefore the hedge ratios affecting the hedging portfolio, are calculated by simulation. Hence, it does seem that the hedging errors are much more sensitive to the simulation errors. Therefore the closed-form BS model has a relative advantage in calculating the hedging errors, as with the simulated models there seems to be very large individual errors[17] that cause also the averages to grow much bigger. All the more so, as in some categories there are only a few observations.

The results seem to be somewhat different for the two data sets, i.e. the two different underlying assets. For the Euro Stoxx 50 the BS model is not really much better than the simulated models, but for the Nokia share there seems to be even more variation in the simulated models' errors causing the aggregate averages growing as well. For example the SV model, the aggregate relative average error of which is some 25% yields average relative errors ranging from 2.26% (moneyness 0.94-0.97, maturity under 60 days, the best fit in the category) to –332.21% (moneyness 0.97-1.00, maturity under 60 days, the worst fit in the category). It is unlikely that differences this big between two neighboring categories can be caused by anything else than random noise in calculating the hedging portfolio. Calculating the portfolio in turn boils down to computing the partial derivatives. This result supports the discussion by e.g. Reiss and Wystup (2001) or Broadie and Glasserman (1996) and suggests that computing the partial derivatives is perhaps the most crucial part in constructing the hedging portfolio. This in turn suggests that the finite difference method, known to be a bit crude by Boyle

---

[17] See appendix D for the graphical representation of the errors.

et al. (1997) but nevertheless applied by us for computing the derivative[18] with respect to the instantaneous volatility, might not be a method accurate enough for practical use or at least the number of simulations would have to be increased substantially. In addition, it is more time-consuming to compute than some direct simulation method in the spirit of Broadie and Glasserman (1996) would be, let alone a closed-form solution whenever available.

But, even if the results are adversely affected by inefficient methods for computing the derivatives, especially the one with respect to the volatility, some information can still be obtained from the comparison. What seems to be true for options on both underlying assets is that the SVT and SVCJ models perform, on average, worse than the other models. Together with the pricing performance results, which pointed to the same direction, it can be inferred that these are the models least likely to represent the actual asset price dynamics of our data. The conclusion concerning the SVCJ model is not really surprising, given that already Eraker et al. (2002) found little evidence supporting this model. However, as was already briefly discussed in section 5.2, the poor performance of the SVT model is a little more surprising. The model has not been studied by many authors, but Liesenfeld and Jung (2000) did report significant improvement over the SV-normal model when using a SVT model. However, their specification of the volatility process was not exactly the same as the modified CIR model used by us, which could account for at least a part in the loss of explanatory power implied by our results. Furthermore, with a different data set the results could be different and it is also possible that there was some estimation error in our parameter estimates causing further inaccuracy in pricing and hedging. On the other hand, Liesenfeld and Jung (2000) only studied the pricing ability of the model and it is possible that including the hedging performance as a metric for the model accuracy would have deteriorated also their results.

Another conclusion to be drawn from the hedging results is that the SVJ model seems to get more support than from the pricing performance-based comparison. Even if there is noise in the SVJ hedging errors as well, they still seem to be closer to zero than for example the SV model's errors. Therefore the conclusion drawn by almost all prior

---

[18] See appendix A.

studies (Bakshi et al. 1997 & 2000, Pan 2002, among many others) that jumps in the price process really do add the explanatory power of a model, gets support also from our results. Also the empirical finding of Eraker et al. (2002), suspected already before that at least by Bates (2000) and Pan (2002), that the true volatility process also has jumps in it, gets some support from our hedging results. However, as already mentioned, this holds only for the SVIJ model, the correlation between the jumps in the two processes does not get support from our results. Measured with the hedging performance this model seems to perform even worse than in terms of the pricing performance. All in all, our hedging results concerning the SVIJ and SVCJ models give further support to the conclusions of Eraker et al. (2002), who only compared the models in terms of pricing ability.

What seems to be true in general also for the hedging performance, is that the errors tend to be bigger for OTM options than for ATM and ITM options. For the Nokia options errors in hedging ATM options are the smallest, whereas for the index options there does not seem to be a big difference between ATM and ITM options. However, for deep OTM index options, the simulated models get relatively much more support in comparison with the BS than what is the case with the other options. Furthermore, based on the index option results, when time to maturity increases, so does the relative performance of the simulated models compared with the BS. In the long-term options even the SVT model, otherwise performing badly, achieves some good results in terms of hedging.

6 Conclusions

It was the goal of this study to compare some of the most important models for the asset price dynamics on European option data. Furthermore, the comparison included also some models that previously have not received widespread attention in the literature. By studying stochastic volatility models with t-distributed innovations in the price process and jump-diffusion models with jumps allowed also in the volatility process, the thesis aimed at exploring whether the more traditional specifications of stochastic volatility and jump-diffusion models are in fact those that best fit the market prices of options, or if they can be improved with these more recently presented modifications to the price dynamics. The comparison was mainly conducted in terms of out-of-sample pricing and hedging performance of the models, although some information could already be extracted from the parameter estimates and the in-sample fit.

What became apparent from the study is that in general the results of the prior American studies get support also from the European derivatives markets. Some differences can of course always be found, but whether the different market, different time period or some other factor causes these, remains unclear. The results suggest that, in accordance to several authors', such as Bakshi et al. (1997, 2000), Bates (1996a, 2000), Andersen et al. (2002) and Pan (2002), results from different time periods, the jump-diffusion models seem to present a more accurate description of the true asset price dynamics than models without jumps. However, the support to jump-diffusion models given by our results was not as strong as that in the previous American studies.

Also jumps in the volatility process, as previously modeled by Duffie et al. (2000) and Eraker et al. (2002), get some support from our results. This evidence is, however, more mixed than that with models with jumps only in the price process. When pricing index options the model with independent jumps in both processes does fit the market data better than any other model, but when pricing stock options and when hedging options with either type of underlying asset, it seems to have trouble outperforming other models. Furthermore, the model with correlated jumps in the two processes does not, in conformity with the results of Eraker et al. (2002), get support from our data.

The stochastic volatility model using the t-distribution does not seem to perform very well according to our data. Even though Liesenfeld and Jung (2000) argue that such a model represents a significant improvement over the SV-model, the results of this thesis do not support this claim. Especially in terms of hedging performance, an issue ignored by Liesenfeld and Jung (2000), the SVT-model seems to underperform all other models. Because of these results we recommend that the SVT model, at least without further amendments, should be regarded as a less valuable model for pricing and especially for hedging options on the market. Another reason for this recommendation is given by the fact that closed-form solutions are not available for the SVT, but can be applied to the other models studied here. A closed-form solution of course implies that the simulation error could be reduced or totally eliminated.

When comparing the performance of the models on pricing and hedging the two different underlying assets studied in this thesis, namely the DJ Euro Stoxx 50 index and Nokia stock, it can be argued that in general the models seem to perform much better when the underlying asset is the index. Hence, the results suggest that not only do the stock options demand more of the pricing model used, but also are they more sensitive to estimation and simulation error. This is implied by the fact that the simulated models seem to perform relatively worse in comparison with the closed-form Black-Scholes model with stock options than with index options.

We also argue that much in the same way as there is a difference in sensitivity to estimation and simulation error in options with different types of underlying assets, also hedging an option demands more of the model and is more sensitive to random errors than merely pricing the option. Reasons for this conclusion are that the BS model again seems to perform relatively better in hedging comparison, whereas Bakshi et al. (1997), who studied exclusively closed-form models, find that in the hedging comparison the deficits of the BS become more significant than in pricing. Furthermore, a single simulation model seems to have much more variation in its performance when hedging options than when pricing them.

Using Monte Carlo simulation it is possible to include also the SVT model in the analysis, and some information about the models' sensitivity to random errors can be obtained. However, the use of simulation does also have its downsides. Most importantly, the comparisons between models and telling whether they truly represent the properties of the true price dynamics do become more difficult when simulation is used. In addition to generating individual large errors blurring the picture, simulation makes estimation of parameters somewhat less reliable. Especially the implied parameter estimation used in the thesis can be sensitive to simulation error. Therefore it is recognized that the results presented in the thesis do not necessarily tell the whole truth and with some other estimation method or perhaps by using closed-form models the results could have been a little different. Whether the use of Monte Carlo simulation truly biased our results in the end, remains for the future research to find out.

Other issues requiring further academic attention are whether our specifications of the models, especially the SVT, SVIJ and SVCJ, were optimal or if their performance could be enhanced by different assumptions. For example, would it improve the SVT model to include t-distributed innovations in the volatility process as well? And concerning the bivariate jump models, the question still remains as to what is the correct economic interpretation of jumps in the volatility process. So far researchers have justified including jumps in volatility by empirical results, but theoretical explanations are still somewhat incomplete, especially as the correlated jumps do not seem to get support. Furthermore, it remains unclear if the differences between stock and index options are valid for all stocks and for all indices or if they are just coincidental and depend on the specific stock or index studied.

7 References

Amin, Kaushik I. (1993) "Jump-Diffusion Option Valuation in Discrete Time" *Journal of Finance*, Vol. 48, No. 5, pp. 1833-1863

Amin, Kaushik I. and Ng, Victor K. (1993) "Option Valuation with Systematic Stochastic Volatility" *Journal of Finance*, Vol. 48, No. 3, pp. 881-910

Andersen, Torben G., Benzoni, Luca and Lund, Jesper (2002) "An Empirical Investigation of Continuous-Time Equity Return Models" *Journal of Finance*, Vol. 57, No. 3, pp. 1239-1284

Andersen, Torben G., Bollerslev, Tim, Diebold, Francis X. and Ebens, Heiko (2001) "The Distribution of Realized Stock Return Volatility" *Journal of Financial Economics*, Vol. 61, No. 1, pp. 43-76

Bailey, Warren and Stulz, René M. (1989) "The Pricing of Stock Index Options in a General Equilibrium Model" *Journal of Financial and Quantitative Analysis,* Vol. 24, No. 1, pp. 1-12

Bakshi, Gurdip, Cao, Charles and Chen, Zhiwu (1997) "Empirical Performance of Alternative Option Pricing Models" *Journal of Finance*, Vol. 52, No. 5, pp. 2003-2049

Bakshi, Gurdip, Cao, Charles and Chen, Zhiwu (2000) "Pricing and Hedging Long-Term Options" *Journal of Econometrics*, Vol. 94, No. 1-2, pp. 277-318

Bates, David S. (1991) "The Crash of '87: Was It Expected? The Evidence from Options Markets" *Journal of Finance*, Vol. 46, No. 3, pp. 1009-1044

Bates, David S. (1996a) "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options" *Review of Financial Studies*, Vol. 9, No. 1, pp. 69-107

Bates, David S. (1996b) "Testing Option Pricing Models", in G.S. Maddala and C.R. Rao (eds.), *Statistical Methods in Finance (Handbook of Statistics,* Vol. 14), Elsevier Science BV, Amsterdam, pp. 567-611

Bates, David S. (2000) "Post-'87 Crash Fears in the S&P 500 Futures Option Market" *Journal of Econometrics*, Vol. 94, No. 1-2, pp. 181-238

Black, Fischer (1976) "Studies in Stock Price Volatility Changes", *Proceedings of the 1976 Meeting of the Business and Economic Statistics Section*, American Statistical Association, pp. 177-181

Black, Fischer and Scholes, Myron (1973) "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy,* Vol. 81, No. 3, pp. 637-654

Bollerslev, Tim (1986) "Generalized Autoregressive Conditional Heteroskedasticity" *Journal of Econometrics*, Vol. 31, pp. 307-327

Bollerslev, Tim, Chou, Ray Y. and Kroner, Kenneth F. (1992) "ARCH Modeling in Finance" *Journal of Econometrics*, Vol. 52, pp. 5-59

Boyle, Phelim P. (1977) "Options: A Monte Carlo Approach" *Journal of Financial Economics,* Vol. 4, No. 3, pp. 323-338

Boyle, Phelim, Broadie, Mark and Glasserman, Paul (1997) "Monte Carlo Methods for Security Pricing" *Journal of Economic Dynamics and Control*, Vol. 21, pp. 1267-1321

Brennan, M. J. (1979) "The Pricing of Contingent Claims in Discrete Time Models" *Journal of Finance*, Vol. 34, No. 1, pp. 53-68

Broadie, Mark and Glasserman, Paul (1996) "Estimating Security Price Derivatives Using Simulation" *Management Science,* Vol. 42, No. 2, pp. 269-285

Campbell, J.Y. and Hentschel, L. (1992) "No News Is Good News: An Asymmetric Model of Changing Volatility of Stock Returns", *Journal of Financial Economics*, Vol. 31, No. 3, pp. 281-318

Campbell, John Y., Lo, Andrew W. and MacKinlay, A. Craig (1997) *The Econometrics of Financial Markets*, Princeton University Press, New Jersey

Chernov, Mikhail, Gallant, A. Ronald, Ghysels, Eric and Tauchen, George (2002) "Alternative Models for Stock Price Dynamics", *Journal of Econometrics*, forthcoming

Clewlow, Les and Strickland, Chris (1998) *Implementing Derivatives Models*, John Wiley & Sons Ltd., West Sussex

Cox, John C., Ingersoll, Jonathan E. Jr. and Ross, Stephen A. (1985) "A Theory of the Term Structure of Interest Rates" *Econometrica*, Vol. 53, no. 2, pp. 385-407

Cox, John C. and Ross, Stephen A. (1976) "The Valuation of Options for Alternative Stochastic Processes" *Journal of Financial Economics*, Vol. 3, pp. 145-166

Cox, John C., Ross, Stephen A. and Rubinstein, Mark (1979) "Option Pricing: A Simplified Approach" *Journal of Financial Economics*, Vol. 7, No. 3, pp. 229-263

Das, S.R. and Sundaram, R.K. (1999) "Of Smiles and Smirks: A Term Structure Perspective" *Journal of Financial and Quantitative Analysis*, Vol. 34, No. 2, pp. 211-239

Derman, Emanuel and Kani, Iraj (1994) "The Volatility Smile and Its Implied Tree" Research Note, Goldman Sachs & Co.

Drost, Feike C., Nijman, Theo E. and Werker, Bas J. M. (1998) "Estimation and Testing in Models Containing Both Jumps and Conditional Heteroscedasticity", *Journal of Business & Economic Statistics*, Vol. 16, No. 2, pp. 237-243

Duan, Jin-Chuan (1995) "The GARCH Option Pricing Model" *Mathematical Finance*, Vol. 5, No. 1, pp. 13-32

Duan, Jin-Chuan (1996a) "A Unified Theory of Option Pricing under Stochastic Volatility from GARCH to Diffusion" Working Paper, Faculty of Management, McGill University

Duan, Jin-Chuan (1996b) "Cracking the Smile" *Risk*, Vol. 9, No. 12, pp. 55-59

Duan, Jin-Chuan, Gauthier, Geneviève and Simonato, Jean-Guy (2001) "Asymptotic Distribution of the EMS Option Price Estimator" *Management Science*, Vol. 47, No. 8, pp. 1122-1132

Duan, Jin-Chuan and Simonato, Jean-Guy (1998) "Empirical Martingale Simulation for Asset Prices" *Management Science*, Vol. 44, No. 9, pp. 1218-1233

Duffie, Darrell (2001) *Dynamic Asset Pricing Theory*, 3rd ed., Princeton University Press, New Jersey

Duffie, Darrell, Pan, Jun and Singleton, Kenneth (2000) "Transform Analysis and Asset Pricing for Affine Jump-Diffusions" *Econometrica*, Vol. 68, No. 6, pp. 1343-1376

Dumas, Bernard, Fleming, Jeff and Whaley, Robert E. (1998) "Implied Volatility Functions: Empirical Tests" *Journal of Finance*, Vol. 53, No. 6, pp. 2059-2106

Engle, Robert F. (1982) "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation" *Econometrica,* Vol. 50, No. 4, pp. 987-1007

Engle, Robert F. and Mustafa, Chowdhury (1992) "Implied ARCH Models from Options Prices" *Journal of Econometrics*, Vol. 52, pp. 289-311

Engle, Robert F. and Ng, Victor K. (1993) "Measuring and Testing the Impact of News on Volatility" *Journal of Finance* Vol. 48, No. 5, pp. 1749-1778

Eraker, Bjørn, Johannes, Michael and Polson, Nicholas (2002) "The Impact of Jumps in Volatility and Returns" *Journal of Finance,* forthcoming in June, 2003

Eurex (2003) www.eurexchange.com, accessed February 12th, 2003

Fama, Eugene F. (1965) "The Behavior of Stock Market Prices" *Journal of Business*, Vol. 38, pp. 34-105

Figlewski, Stephen and Wang, Xiaozu (2000) "Is the "Leverage Effect" a Leverage Effect?" Working Paper, NYU Stern School of Business

Ghysels, Eric, Harvey, Andrew and Renault, Éric (1996) "Stochastic Volatility" in G.S. Maddala and C.R. Rao (eds.), *Statistical Methods in Finance (Handbook of Statistics,* Vol. 14), Elsevier Science BV, Amsterdam, pp. 119-191

Glasserman, Paul and Zhao, Xiaoliang (1999) "Fast Greeks in Forward Libor Models" *Journal of Computational Finance*, Vol 3, No. 1, pp. 5-39

Greene, William H. (2000) *Econometric Analysis*, 4th ed., Prentice-Hall Inc., New Jersey

Hagan, Patrick S., Kumar, Deep, Lesniewski, Andrew S. and Woodward, Diana E. (2002) "Managing Smile Risk" *Wilmott Magazine*, September Issue, pp. 84-108

Harrison, J. Michael and Kreps, David M. (1979) "Martingales and Arbitrage in Multiperiod Securities Markets" *Journal of Economic Theory*, Vol. 20, pp. 381-408

Heston, Stephen L. (1993) "A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options" *Review of Financial Studies*, Vol. 6, No. 2, pp. 327-343

Heston, Stephen L. and Nandi, Saikat (2000) "A Closed-Form GARCH Option Valuation Model" *Review of Financial Studies,* Vol. 13, No. 3, pp. 585-625

Hull, John and White, Alan (1987) "The Pricing of Options with Stochastic Volatilities" *Journal of Finance*, Vol. 42, No. 2, pp. 281-300

Jarrow, Robert A. and Turnbull, Stuart M. (2000) *Derivative Securities*, 2nd ed., South-Western College Publishing, USA

Johnson, Simon and Lee, Han (2003) "Capturing the Smile" *Risk*, Vol. 16, No. 3, pp. 89-93

Johnson, Herb and Shanno, David (1987) "Option Pricing when the Variance Is Changing" *Journal of Financial and Quantitative Analysis*, Vol. 22, No. 2, pp. 143-151

Kallsen, Jan and Taqqu, Murad S. (1998) "Option Pricing in ARCH-Type Models" *Mathematical Finance*, Vol. 8, No. 1, pp. 13-26

Liesenfeld, Roman and Jung, Robert C. (2000) "Stochastic Volatility Models: Conditional Normality Versus Heavy-Tailed Distributions" *Journal of Applied Econometrics*, Vol. 15, No. 2, pp. 137-160

Luenberger, David G. (1998) *Investment Science*, Oxford University Press Inc., New York

Mandelbrot, Benoit (1963) "The Variation of Certain Speculative Prices" *Journal of Business*, Vol. 36, pp. 394-419

Melino, A. and Turnbull, S. M. (1990) "Pricing Foreign Currency Options with Stochastic Volatility" *Journal of Econometrics*, Vol. 45, pp. 239-265

Merton, Robert C. (1973) "Theory of Rational Option Pricing" *Bell Journal of Economics and Management Science*, Vol. 4, pp. 141-183

Merton, Robert C. (1976) "Option Pricing when Underlying Stock Returns Are Discontinuous", *Journal of Financial Economics*, Vol. 3, pp. 125-144

Merville, Larry J. and Pieptea, Dan R. (1989) "Stock-Price Volatility, Mean-Reverting Diffusion, and Noise" *Journal of Financial Economics,* Vol. 24, pp. 193-214

Nandi, Saikat (1998) "How Important Is the Correlation Between Returns and Volatility in a Stochastic Volatility Model? Empirical Evidence from Pricing and Hedging in the S&P 500 Index Options Market" *Journal of Banking & Finance*, Vol. 22, No. 5, pp. 589-610

Nelson, Daniel B. (1990) "ARCH Models as Diffusion Approximations" *Journal of Econometrics* Vol. 45, pp. 7-38

Nelson, Daniel B. (1991) "Conditional Heteroskedasticity in Asset Returns: A New Approach" *Econometrica*, Vol. 59, No. 2, pp. 347-370

Nokia (2003) www.nokia.com, accessed January 13th, 2003

Pan, Jun (2002) "The Jump-Risk Premia Implicit in Options: Evidence from an Integrated Time-Series Study" *Journal of Financial Economics*, Vol. 63, No. 1, pp. 3-50

Reiss, Oliver and Wystup, Uwe (2001) "Computing Option Price Sensitivities Using Homogeneity and Other Tricks" *Journal of Derivatives*, Vol. 9, No. 2, pp. 41-53

R Homepage (2003) www.r-project.org, accessed March 4th, 2003

Ritchken, Peter and Trevor, Rob (1999) "Pricing Options under Generalized GARCH and Stochastic Volatility Processes" *Journal of Finance*, Vol. 54, No. 1, pp. 377-402

Ross, Stephen A. (1978) "A Simple Approach to the Valuation of Risky Streams" *Journal of Business*, Vol. 51, pp. 453-475

Rubinstein, Mark (1976) "The Valuation of Uncertain Income Streams and the Pricing of Options" *Bell Journal of Economics and Management Science*, Vol. 7, No. 2, pp. 407-425

Rubinstein, Mark (1985) "Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978" *Journal of Finance*, Vol. 40, No. 2, pp. 455-480

Rubinstein, Mark (1994) "Implied Binomial Trees" *Journal of Finance*, Vol. 49, No. 3, pp. 771-818

Schmitt, Christian (1996) "Option Pricing Using EGARCH Models", Centre for European Economic Research (ZEW) Discussion Paper No. 96-20, Mannheim.

Schwert, G. William (1989) "Why Does Stock Market Volatility Change over Time?" *Journal of Finance*, Vol. 44, No. 5, pp. 1115-1153

Scott, Louis O. (1997) "Pricing Stock Options in a Jump-Diffusion Model with Stochastic Volatility and Interest Rates: Applications of Fourier Inversion Methods" *Mathematical Finance,* Vol. 7, No. 4, pp. 413-426

Stein, Elias M. and Stein, Jeremy C. (1991) "Stock Price Distributions with Stochastic Volatility: An Analytic Approach" *Review of Financial Studies*, Vol. 4, No. 4, pp. 727-752

STOXX Limited (2003) www.stoxx.com, accessed February 12th, 2003

Taylor, Stephen J. (1994) "Modeling Stochastic Volatility: A Review and Comparative Study" *Mathematical Finance*, Vol. 4, No. 2, pp. 183-204

Wiggins, James B. (1987) "Option Values Under Stochastic Volatility: Theory and Empirical Estimates" *Journal of Financial Economics*, Vol. 19, pp. 351-372

8 Appendices

Appendix A: Partial Derivatives of the Option Price

In this appendix we will present the computing methods for partial derivatives of the option price with respect to the initial asset price S_0 and initial volatility V_0.

The general bivariate jump-diffusion model (Duffie et al. 2000, Eraker et al. 2002)

$$d\begin{pmatrix} Y_t \\ V_t \end{pmatrix} = \begin{pmatrix} r - \frac{1}{2}V_t - \lambda_y \mu_y \\ \kappa(\theta - V_t) \end{pmatrix} dt + \sqrt{V_t}\begin{pmatrix} 1 & 0 \\ \rho\sigma_v & \sqrt{1-\rho^2}\sigma_v \end{pmatrix} dW_t + k dq_t \quad \text{(A.1)}$$

where

$Y_t = \log S_t$, W_t is a standard two-dimensional Brownian motion, $q_t = \begin{bmatrix} q_{t,y} & q_{t,v} \end{bmatrix}^T$ is a pair of (potentially equal) Poisson processes with constant intensities $\lambda = \begin{bmatrix} \lambda_y & \lambda_v \end{bmatrix}^T$ and jump sizes $k = \begin{bmatrix} k_y & k_v \end{bmatrix}^T$, the expected jump sizes being μ_y for the price process (k_y normally distributed) and $1/\mu_v$ for the volatility process (k_v exponentially distributed). The correlation coefficient between $W_{t,y}$ and $W_{t,v}$ is denoted by ρ. Discretizing the continuous-time model above, dt is the length of a time step, defined

$$dt \equiv \frac{T}{n}, \quad \text{(A.2)}$$

where T is time to maturity and n is the number of time steps. Then define the discounted payoff of option as

$$P = e^{-rT}\max(S_T - K, 0), \text{ so that the call price } C = E(P). \quad \text{(A.3)}$$

Based on the discrete-time model above,

$$\begin{aligned}
Y_T &= \left(r - \frac{V_{T-1}}{2} - \lambda_y \mu_y \right) dt + \sqrt{dt}\sqrt{V_{T-1}}\, dW_{T-1,y} + k_{T-1,Y}\, dq_{T-1,y} + Y_{T-1} \\
&= \sum_{t=1}^{n} \left(r - \frac{V_{t-1}}{2} - \lambda_y \mu_y \right) dt + \sum_{t=1}^{n} \sqrt{dt}\sqrt{V_{t-1}}\, dW_{t-1,y} + \sum_{t=1}^{n} k_{t-1,Y}\, dq_{t-1,y} \\
&= \left(r - \frac{\overline{V}_t}{2} - \lambda_y \mu_y \right) T + \sum_{t=1}^{n} \sqrt{V_{t-1}}\, dW_{t-1,y} + \sum_{t=1}^{n} k_{t-1,y}\, dq_{t-1,y} \qquad \text{(A.4)}
\end{aligned}$$

and therefore

$$S_T = S_0 \exp\left[\left(r - \frac{\overline{V}}{2} - \lambda_y \mu_y \right) T + \sum_{t=0}^{n-1} \sqrt{V_t}\, dW_{t,y} + \sum_{t=0}^{n-1} k_{t,y}\, dq_{t,y} \right]. \qquad \text{(A.5)}$$

We denote the partial derivative of the call price with respect to the initial asset price Δ_S, i.e.

$$\Delta_S \equiv \frac{\partial C}{\partial S_0} = E\left[\frac{\partial P}{\partial S_0} \right] \qquad \text{(A.6)}$$

(See Broadie & Glasserman, 1996, pp. 280-281, Appendix A, proposition 1 for the proof of the validity of the expectation equation above.) Following Broadie and Glasserman, we calculate the Δ_S by their pathwise method.

$$\frac{\partial P}{\partial S_0} = \frac{\partial P}{\partial S_T} \frac{\partial S_T}{\partial S_0} \qquad \text{(A.7)}$$

First we calculate $\frac{\partial P}{\partial S_T}$. Consider the effect on P a small change in S_T will have. If $S_T \geq K$, the option is in-the-money and any increase Δ in S_T will translate into an increase $e^{-rT}\Delta$ in P. If $S < K$, $P = 0$, and no change in S_T will cause P deviate from zero, so the change in P is zero. Thus, we can write

$$\frac{\partial P}{\partial S_T} = e^{-rT} 1_{[S_T \geq K]}, \qquad \text{(A.8)}$$

in which $1_{[\bullet]}$ denotes the indicator of whether the event in brackets takes place. This result does not hold if S=K, but as the probability of this happening is zero, we ignore this possibility.

Derivating (A.5), we see that

$$\frac{\partial S_T}{\partial S_0} = e^{(r-\bar{V}_t/2)T+\sum\sqrt{V_t}dW_{t,y}+\sum k_Y dq_{t,y}} = \frac{S_T}{S_0} \tag{A.9}$$

Combining the equations (A.6) to (A.9) we get the following expression:

$$\Delta_S = e^{-rT} 1_{[S_T \geq K]} \frac{E[S_T]}{S_0} \tag{A.10}$$

This equation is particularly practical, because with it the derivative can be calculated directly with a single simulation run.

However, calculating the partial derivative with respect to the initial volatility, often also known as *vega* and which we denote by Δ_V, is slightly more complicated. The same pathwise method could again be applied, but this would be considerably more difficult as the different specifications for both the price and the volatility process would directly affect the derivative, so we might end up with a different formula for each model. To make things easy, we approximate the derivative linearly with the *finite -difference* method (see e.g. Boyle et al. 1997, pp. 1303-1306).

In finite-difference approximation the idea is, in short, to calculate the simulated option price for initial volatilities V_0 and $V_0+\varepsilon$,and using the call prices $C(V_0)$ and $C(V_0+\varepsilon)$ approximate the Δ_V by

$$\Delta_V = \frac{C(V_0+\varepsilon)-C(V_0)}{\varepsilon} \tag{A.10}$$

In this study, a value of $\frac{V_0}{10}$ is used as ε.

Appendix B: Example of Hedging an Option with Both Price and Volatility Risk

In this appendix an example of delta-neutral hedging in terms of Bakshi et al. (1997) is presented. The technique is discussed in section 4.4 and the hedging errors calculated from the data are presented in section 5.3, and graphically in appendix D.

Consider an option on Nokia stock, price quotes from September 12, 2000. On that day the Nokia stock price was 50.04 euros. The next trading day, i.e. the day when the error was calculated, was September 13. Next the properties of the two options, the one hedged ("Option Hedged" and the one used in hedging ("Hedging Option"), are presented.

	Option Hedged:	Hedging Option:
Strike price (€)	52.5	50
Moneyness S/K	0.9531	1.0008
Time to maturity	94 days (0.2574 years)	94 days (0.2574 years)
Δ_S	0.508195285	0.572794520
Δ_V	0.02379949	0.0161406

The hedging portfolio consists of X_S units of the underlying stock, X_C units of the hedging option and X_0 units of cash, where the X:s are calculated as in section 4.4 and get the following values:

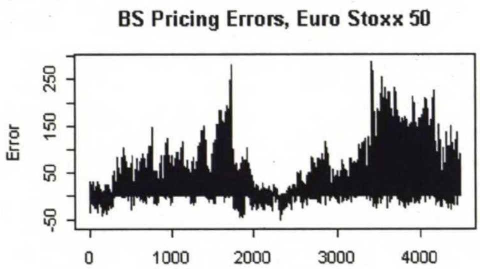
X_S	-0.3364
X_C	1.474511
X_0	13.26655

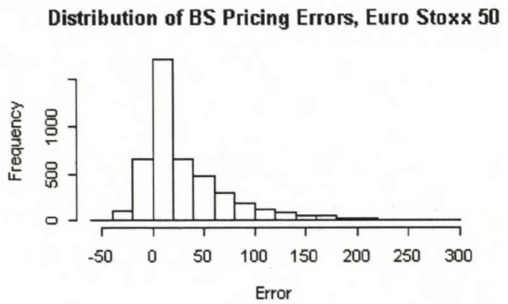
Below are depicted the prices of the two options and of the underlying stock on September 12th and 13th (in euros). The stock price is the one implied by the put-call parity (see section 4.1). Interest rate on the 12th was 4.81% pa.

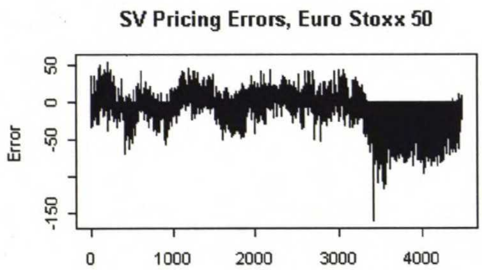
	September 12th	September 13th
Option Hedged	4.44	4.35
Nokia Stock	50.04	49.83
X_S*S	-16.83	-16.76
Hedging Option $C(\widetilde{K})$	5.43	5.36
$X_C * C(\widetilde{K})$	8.01	7.90
Cash Position	13.27	13.43
Hedging Portfolio	4.44	4.41
Error	**0**	**-0.056**
Percentage Error	**0%**	**-1.31%**

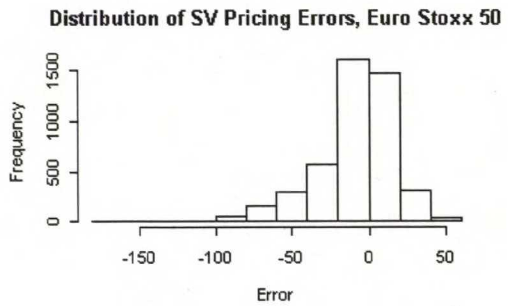
Appendix C: Graphical Representation of the Pricing Errors

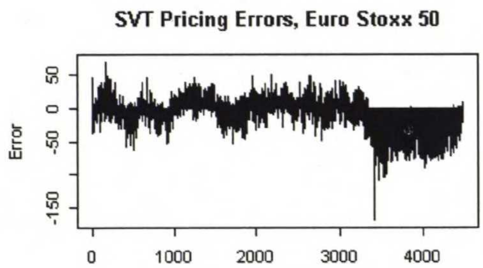
In this appendix the plots and histograms of the absolute pricing errors of different pricing models are presented both for the index and the Nokia stock.

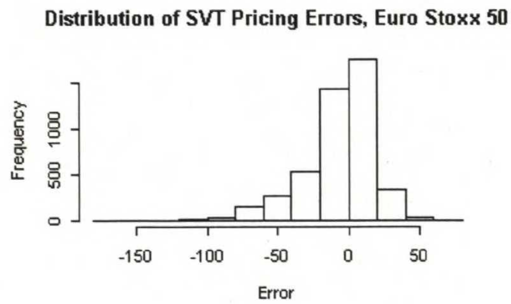


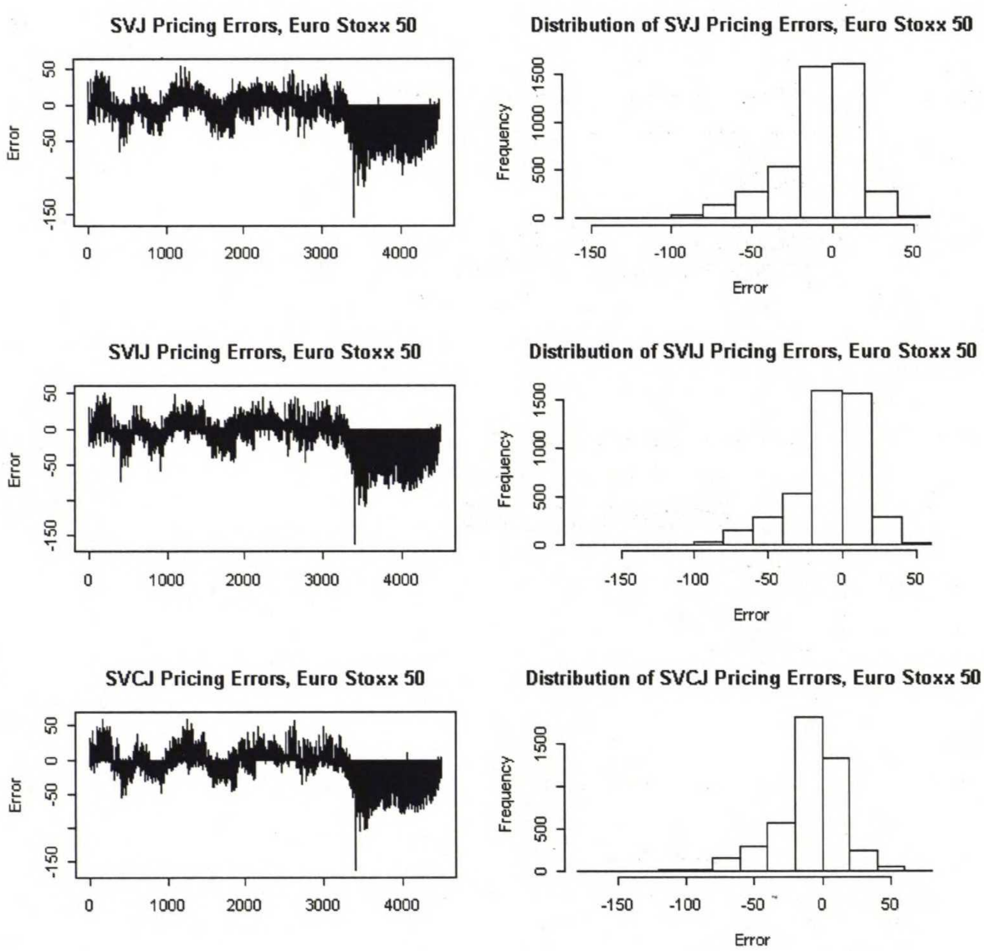


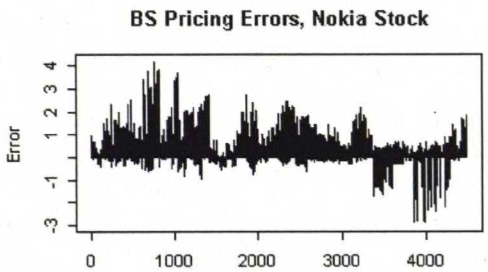


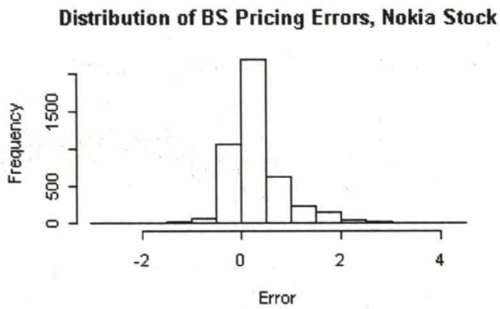


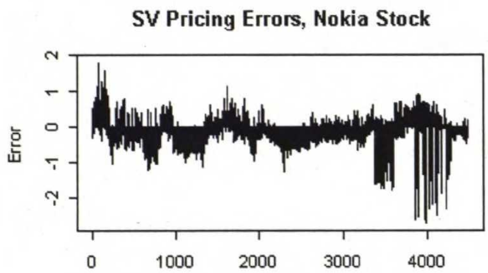


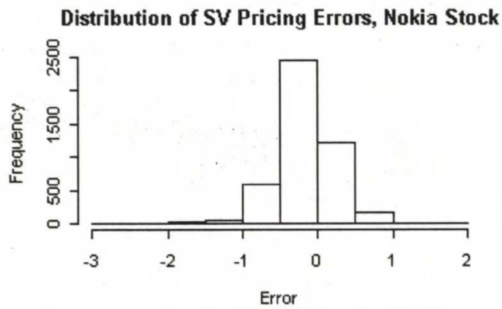


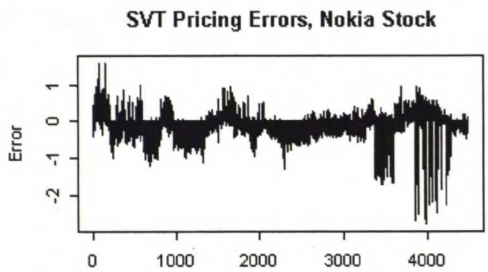

SVJ Pricing Errors, Euro Stoxx 50
Error
Distribution of SVJ Pricing Errors, Euro Stoxx 50
Frequency
Error
SVIJ Pricing Errors, Euro Stoxx 50
Error
Distribution of SVIJ Pricing Errors, Euro Stoxx 50
Frequency
Error
SVCJ Pricing Errors, Euro Stoxx 50
Error
Distribution of SVCJ Pricing Errors, Euro Stoxx 50
Frequency
Error

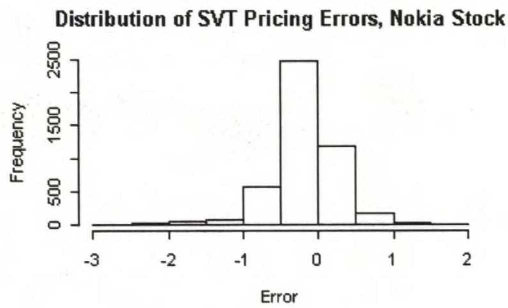

BS Pricing Errors, Nokia Stock
Error
0
1000
2000
3000
4000

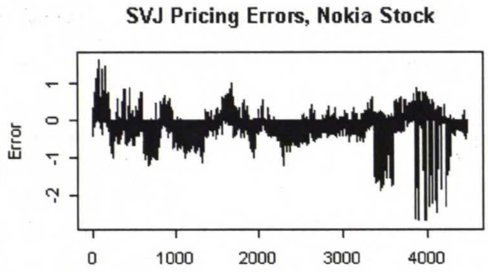

Distribution of BS Pricing Errors, Nokia Stock
Frequency
Error

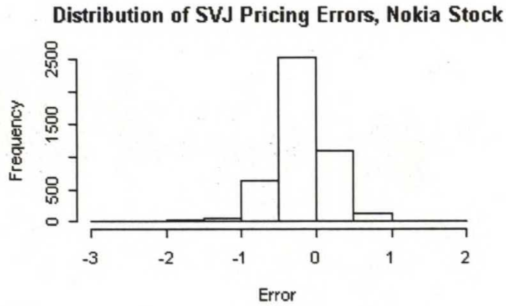

SV Pricing Errors, Nokia Stock
Error
0
1000
2000
3000
4000

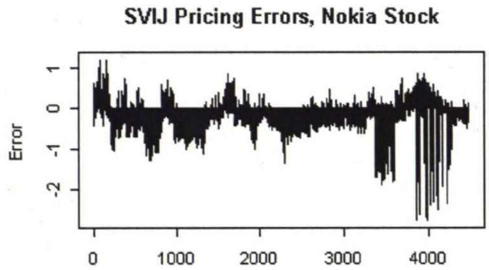

Distribution of SV Pricing Errors, Nokia Stock
Frequency
Error

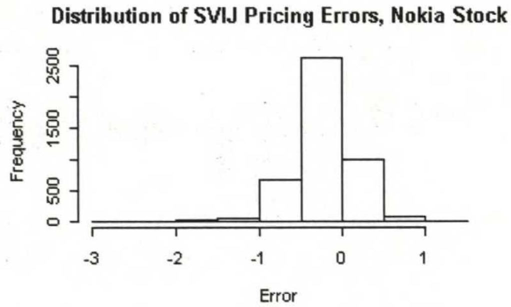

SVT Pricing Errors, Nokia Stock
Error
0
1000
2000
3000
4000

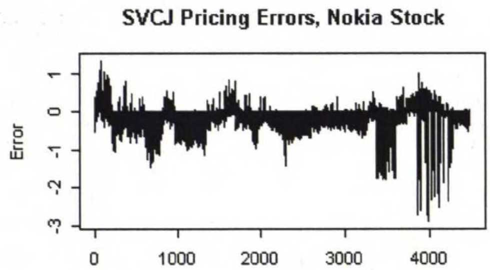

Distribution of SVT Pricing Errors, Nokia Stock
Frequency
Error

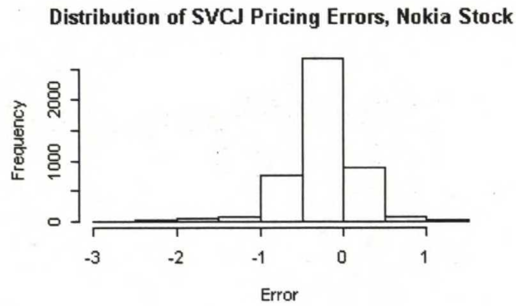

SVJ Pricing Errors, Nokia Stock
Error
1
0
-1
-2
0
1000
2000
3000
4000


Distribution of SVJ Pricing Errors, Nokia Stock
Frequency
2500
1500
500
0
-3
-2
-1
0
1
2
Error


SVIJ Pricing Errors, Nokia Stock
Error
1
0
-1
-2
0
1000
2000
3000
4000

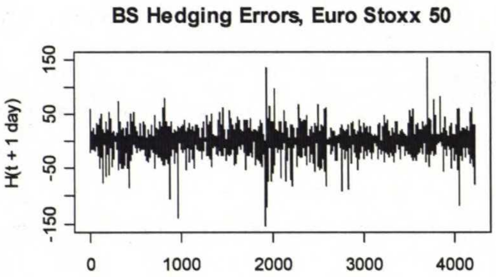

Distribution of SVIJ Pricing Errors, Nokia Stock
Frequency
2500
1500
500
0
-3
-2
-1
0
1
Error

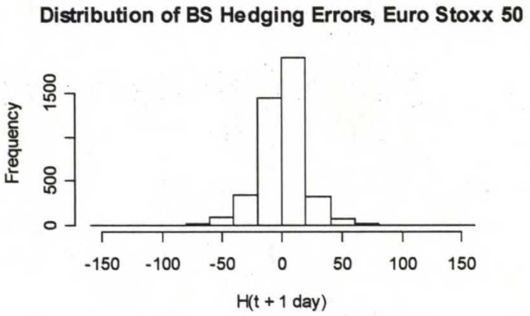

SVCJ Pricing Errors, Nokia Stock
Error
1
0
-1
-2
-3
0
1000
2000
3000
4000

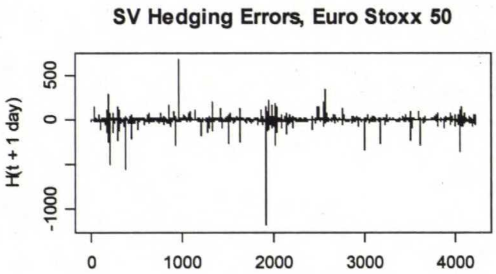

Distribution of SVCJ Pricing Errors, Nokia Stock
Frequency
2000
1000
0
-3
-2
-1
0
1
Error

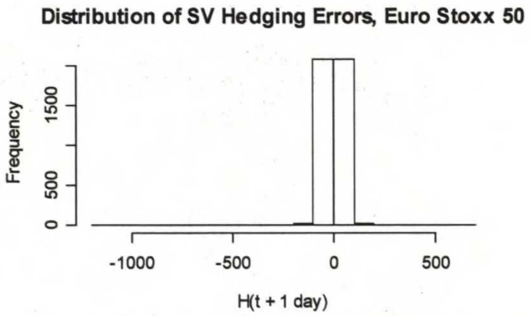
Appendix D: Graphical Representations of the Hedging Errors

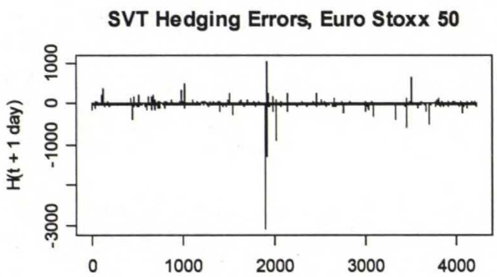
In Appendix C plots and histograms of the absolute hedging errors H(t+Δt) of different pricing models are presented both for the index and the Nokia stock. In our setting Δt is one day.

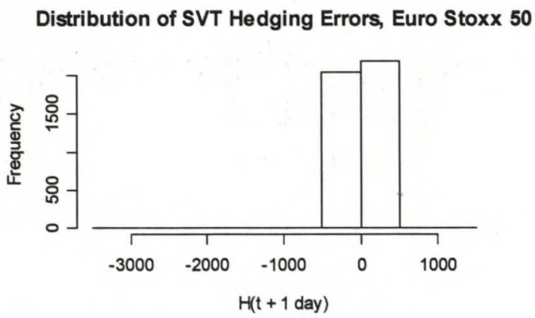


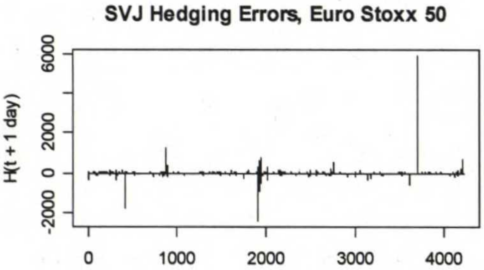


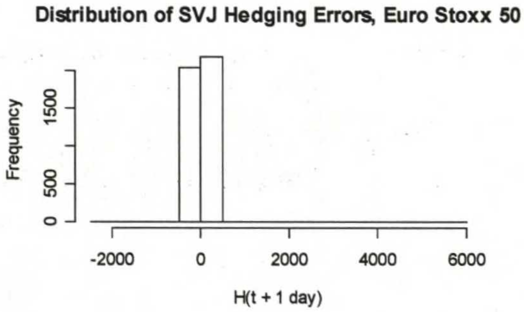


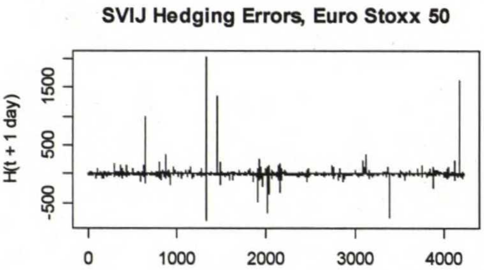


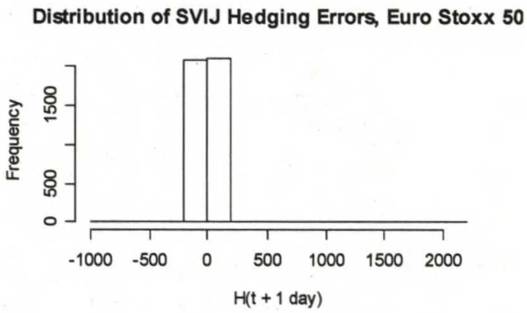


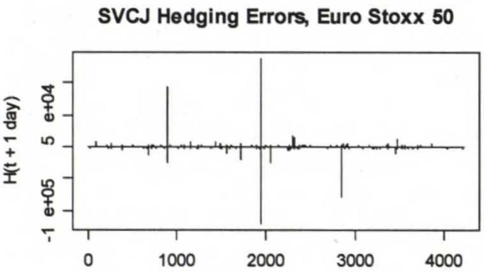


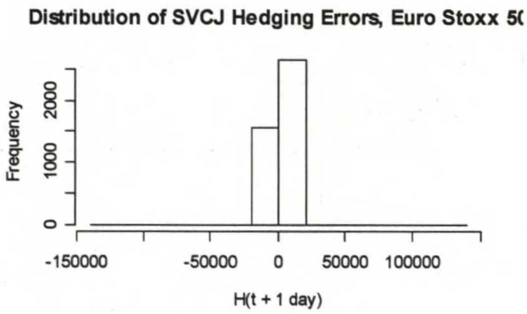

SVJ Hedging Errors, Euro Stoxx 50
H(t + 1 day)
-2000
0
2000
6000
0
1000
2000
3000
4000

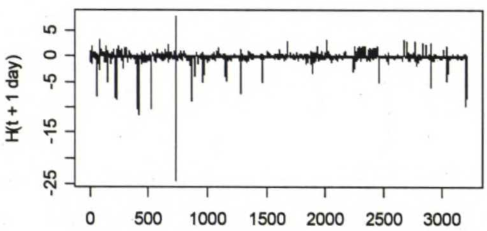

Distribution of SVJ Hedging Errors, Euro Stoxx 50
Frequency
0
500
1500
-2000
0
2000
4000
6000
H(t + 1 day)


SVIJ Hedging Errors, Euro Stoxx 50
H(t + 1 day)
-500
500
1500
0
1000
2000
3000
4000

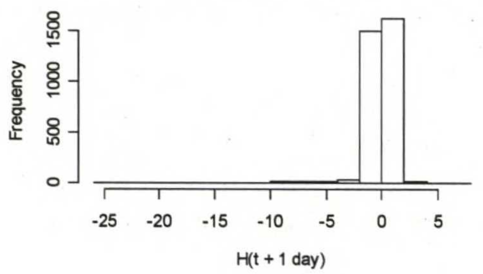

Distribution of SVIJ Hedging Errors, Euro Stoxx 50
Frequency
0
500
1500
-1000
-500
0
500
1000
1500
2000
H(t + 1 day)


SVCJ Hedging Errors, Euro Stoxx 50
H(t + 1 day)
-1 e+05
5
e+04
0
1000
2000
3000
4000

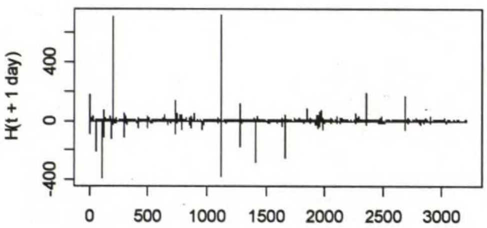

Distribution of SVCJ Hedging Errors, Euro Stoxx 5(
Frequency
0
1000
2000
-150000
-50000
0
50000
100000
H(t + 1 day)



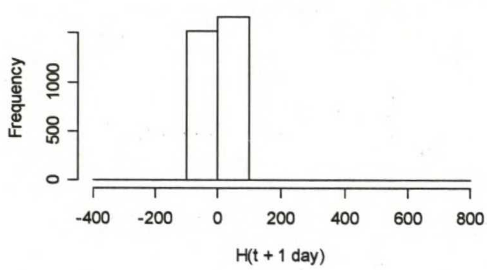
BS Hedging Errors, Nokia Stock



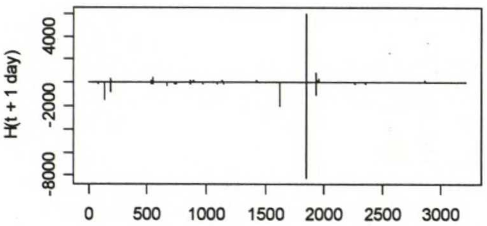
Distribution of BS Hedging Errors, Nokia Stock



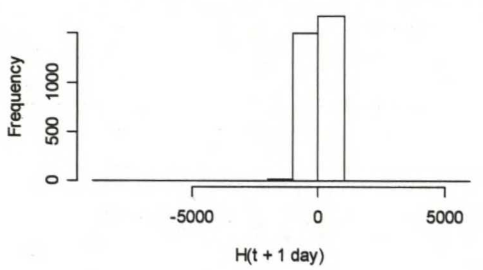
SV Hedging Errors, Nokia Stock



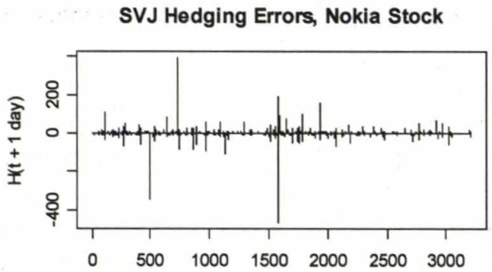
Distribution of SV Hedging Errors, Nokia Stock

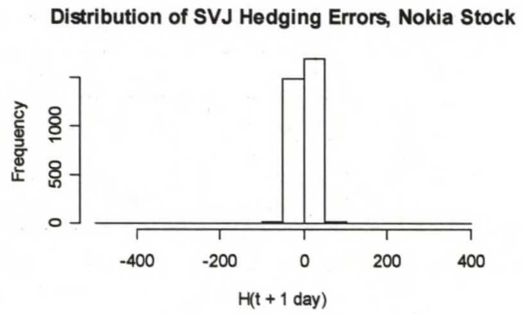


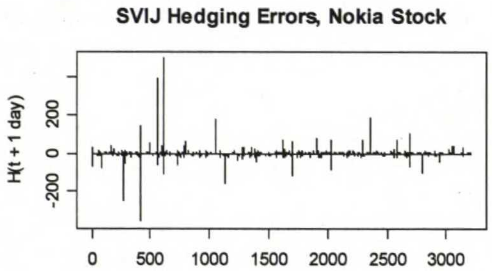
SVT Hedging Errors, Nokia Stock

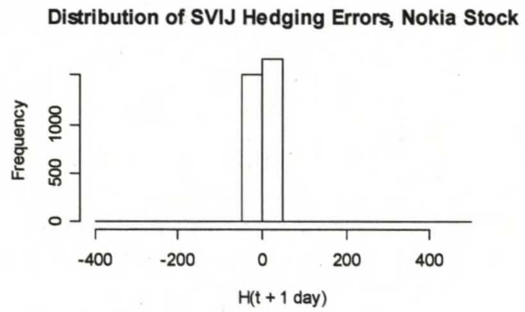


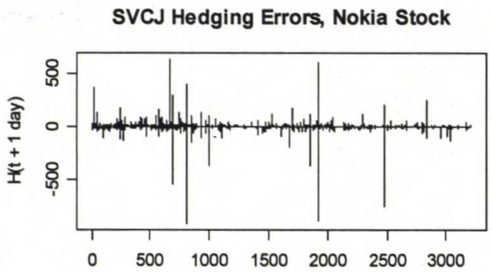
Distribution of SVT Hedging Errors, Nokia Stock

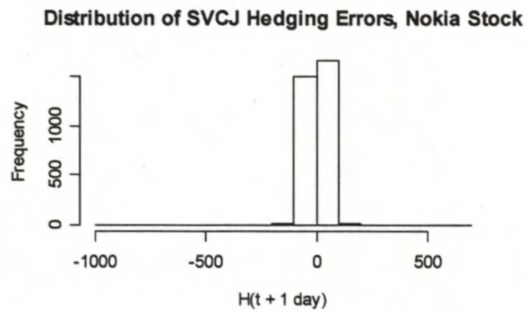

SVJ Hedging Errors, Nokia Stock
H(t + 1 day)
200
0
-400
0
500
1000
1500
2000
2500
3000


Distribution of SVJ Hedging Errors, Nokia Stock
Frequency
0
500
1000
-400
-200
0
200
400
H(t + 1 day)


SVIJ Hedging Errors, Nokia Stock
H(t + 1 day)
200
0
-200
0
500
1000
1500
2000
2500
3000


Distribution of SVIJ Hedging Errors, Nokia Stock
Frequency
0
500
1000
-400
-200
0
200
400
H(t + 1 day)


SVCJ Hedging Errors, Nokia Stock
H(t + 1 day)
500
0
-500
0
500
1000
1500
2000
2500
3000


Distribution of SVCJ Hedging Errors, Nokia Stock
Frequency
0
500
1000
-1000
-500
0
500
H(t + 1 day)